CHAPTER 6

CONTROL SYSTEMS

YEAR 2012 **TWO MARKS**

MCQ 6.1 The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

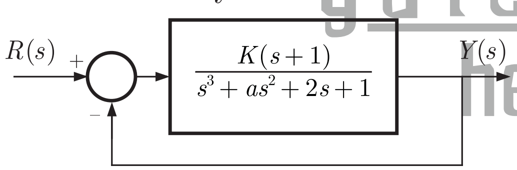
$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

(A) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$ (B) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(C) $a_1 = 0, a_3 \neq 0, a_3 = 0$ (D) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

MCQ 6.2 The feedback system shown below oscillates at 2 rad/s when



(A) $K = 2$ and $a = 0.75$ (B) $K = 3$ and $a = 0.75$

(C) $K = 4$ and $a = 0.5$ (D) $K = 2$ and $a = 0.5$

Statement for Linked Answer Questions 3 and 4 :

The transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

MCQ 6.3 $G_c(s)$ is a lead compensator if

(A) $a = 1, b = 2$ (B) $a = 3, b = 2$

(C) $a = -3, b = -1$ (D) $a = 3, b = 1$

MCQ 6.4 The phase of the above lead compensator is maximum at

(A) $\sqrt{2}$ rad/s (B) $\sqrt{3}$ rad/s

(C) $\sqrt{6}$ rad/s (D) $1/\sqrt{3}$ rad/s

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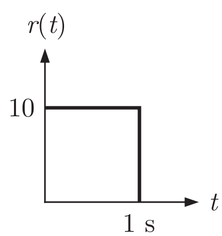
YEAR 2011 ONE MARK

MCQ 6.5 The frequency response of a linear system $G(j\omega)$ is provided in the tubular form below

$\lvert G(j\omega)\rvert$	1.3	1.2	1.0	0.8	0.5	0.3
$\angle G(j\omega)$	-130°	-140°	-150°	-160°	-180°	-200°

(A) 6 dB and 30°
(B) 6 dB and -30°
(C) -6 dB and 30°
(D) -6 dB and -30°

MCQ 6.6 The steady state error of a unity feedback linear system for a unit step input is 0.1. The steady state error of the same system, for a pulse input $r(t)$ having a magnitude of 10 and a duration of one second, as shown in the figure is



(A) 0
(B) 0.1
(C) 1
(D) 10

MCQ 6.7 An open loop system represented by the transfer function

$$G(s) = \frac{(s-1)}{(s+2)(s+3)} \text{ is}$$

(A) Stable and of the minimum phase type
(B) Stable and of the non–minimum phase type
(C) Unstable and of the minimum phase type
(D) Unstable and of non–minimum phase type

YEAR 2011 TWO MARKS

MCQ 6.8 The open loop transfer function $G(s)$ of a unity feedback control system is given as

$$G(s) = \frac{K\left(s+\frac{2}{3}\right)}{s^2(s+2)}$$

From the root locus, at can be inferred that when K tends to positive infinity,

(A) Three roots with nearly equal real parts exist on the left half of the s

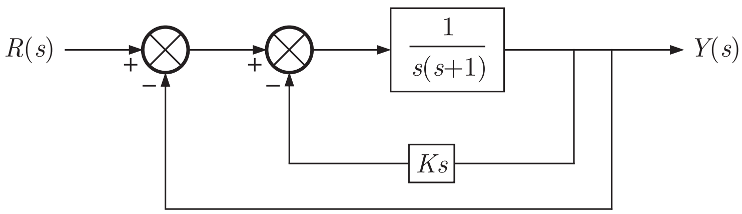
-plane

(B) One real root is found on the right half of the s-plane

(C) The root loci cross the $j\omega$ axis for a finite value of $K; K \neq 0$

(D) Three real roots are found on the right half of the s-plane

MCQ 6.9 A two loop position control system is shown below



The gain K of the Tacho-generator influences mainly the

(A) Peak overshoot

(B) Natural frequency of oscillation

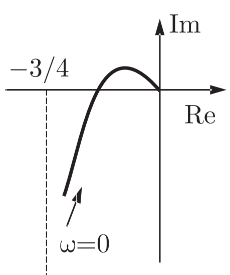
(C) Phase shift of the closed loop transfer function at very low frequencies $(\omega \to 0)$

(D) Phase shift of the closed loop transfer function at very high frequencies $(\omega \to \infty)$

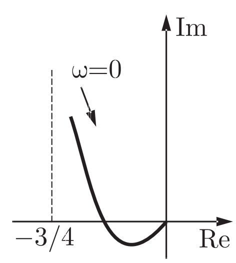
YEAR 2010 TWO MARKS

MCQ 6.10 The frequency response of $G(s) = \frac{1}{s(s+1)(s+2)}$ plotted in the complex $G(j\omega)$ plane (for $0 < \omega < \infty$) is

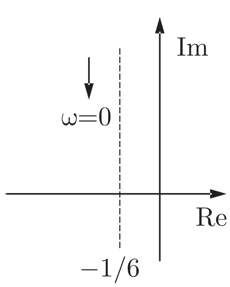
(A)



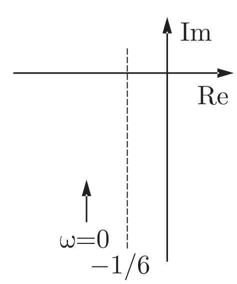
(B)



(C)



(D)



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MCQ 6.11 The system $\dot{\boldsymbol{X}} = A\boldsymbol{X} + B\boldsymbol{u}$ with $A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is

(A) Stable and controllable

(B) Stable but uncontrollable

(C) Unstable but controllable

(D) Unstable and uncontrollable

MCQ 6.12 The characteristic equation of a closed-loop system is $s(s+1)(s+3)k(s+2) = 0, k > 0$. Which of the following statements is true ?

(A) Its root are always real

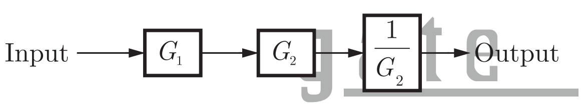
(B) It cannot have a breakaway point in the range $-1 < \text{Re}[s] < 0$

(C) Two of its roots tend to infinity along the asymptotes $\text{Re}[s] = -1$

(D) It may have complex roots in the right half plane.

YEAR 2009 **ONE MARK**

MCQ 6.13 The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as $G_1, G_2, 1/G_3$. The relative small errors associated with each respective subsystem G_1, G_2 and G_3 are $\varepsilon_1, \varepsilon_2$ and ε_3. The error associated with the output is :



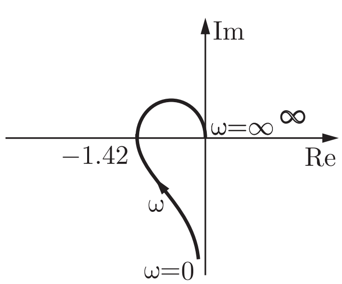
(A) $\varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3}$

(B) $\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_3}$

(C) $\varepsilon_1 + \varepsilon_2 - \varepsilon_3$

(D) $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$

MCQ 6.14 The polar plot of an open loop stable system is shown below. The closed loop system is



(A) always stable

(B) marginally stable

(C) un-stable with one pole on the RH s-plane

(D) un-stable with two poles on the RH s-plane

MCQ 6.15 The first two rows of Routh's tabulation of a third order equation are as follows.

s^3 2 2
s^2 4 4

This means there are

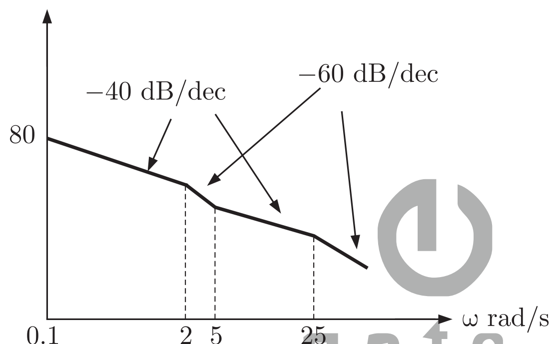
(A) Two roots at $s = \pm j$ and one root in right half s-plane

(B) Two roots at $s = \pm j2$ and one root in left half s-plane

(C) Two roots at $s = \pm j2$ and one root in right half s-plane

(D) Two roots at $s = \pm j$ and one root in left half s-plane

MCQ 6.16 The asymptotic approximation of the log-magnitude v/s frequency plot of a system containing only real poles and zeros is shown. Its transfer function is

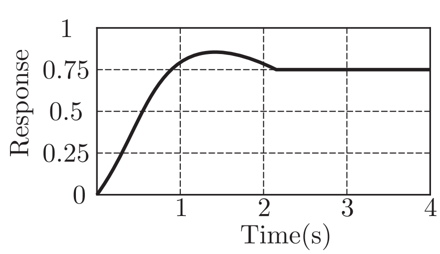


(A) $\dfrac{10(s+5)}{s(s+2)(s+25)}$ (B) $\dfrac{1000(s+5)}{s^2(s+2)(s+25)}$

(C) $\dfrac{100(s+5)}{s(s+2)(s+25)}$ (D) $\dfrac{80(s+5)}{s^2(s+2)(s+25)}$

YEAR 2009 TWO MARKS

MCQ 6.17 The unit-step response of a unity feed back system with open loop transfer function $G(s) = K/((s+1)(s+2))$ is shown in the figure.
The value of K is



(A) 0.5 (B) 2

(C) 4 (D) 6

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MCQ 6.18 The open loop transfer function of a unity feed back system is given by $G(s) = (e^{-0.1s})/s$. The gain margin of the is system is

(A) 11.95 dB (B) 17.67 dB

(C) 21.33 dB (D) 23.9 dB

Common Data for Question 19 and 20 :

A system is described by the following state and output equations

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

$$y(t) = x_1(t)$$

when $u(t)$ is the input and $y(t)$ is the output

MCQ 6.19 The system transfer function is

(A) $\frac{s+2}{s^2+5s-6}$ (B) $\frac{s+3}{s^2+5s+6}$

(C) $\frac{2s+5}{s^2+5s+6}$ (D) $\frac{2s-5}{s^2+5s-6}$

MCQ 6.20 The state-transition matrix of the above system is

(A) $\begin{bmatrix} e^{-3t} & 0 \\ e^{-2t}+e^{-3t} & e^{-2t} \end{bmatrix}$ (B) $\begin{bmatrix} e^{-3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

(C) $\begin{bmatrix} e^{-3t} & e^{-2t}+e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$ (D) $\begin{bmatrix} e^{3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

YEAR 2008 ONE MARK

MCQ 6.21 A function $y(t)$ satisfies the following differential equation :

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

where $\delta(t)$ is the delta function. Assuming zero initial condition, and denoting the unit step function by $u(t), y(t)$ can be of the form

(A) e^t (B) e^{-t}

(C) $e^t u(t)$ (D) $e^{-t} u(t)$

YEAR 2008 TWO MARK

MCQ 6.22 The transfer function of a linear time invariant system is given as

$$G(s) = \frac{1}{s^2+3s+2}$$

The steady state value of the output of the system for a unit impulse input applied at time instant $t = 1$ will be

(A) 0 (B) 0.5

(C) 1 (D) 2

MCQ 6.23 The transfer functions of two compensators are given below :

$$C_1 = \frac{10(s+1)}{(s+10)}, \quad C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct ?

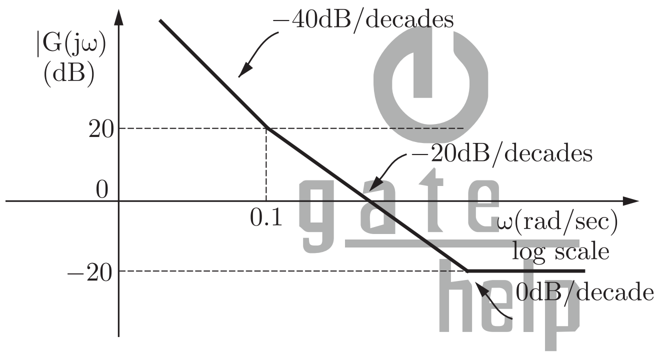
(A) C_1 is lead compensator and C_2 is a lag compensator

(B) C_1 is a lag compensator and C_2 is a lead compensator

(C) Both C_1 and C_2 are lead compensator

(D) Both C_1 and C_2 are lag compensator

MCQ 6.24 The asymptotic Bode magnitude plot of a minimum phase transfer function is shown in the figure :

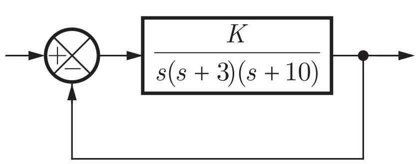


This transfer function has

(A) Three poles and one zero (B) Two poles and one zero

(C) Two poles and two zero (D) One pole and two zeros

MCQ 6.25 Figure shows a feedback system where $K > 0$



The range of K for which the system is stable will be given by

(A) $0 < K < 30$ (B) $0 < K < 39$

(C) $0 < K < 390$ (D) $K > 390$

MCQ 6.26 The transfer function of a system is given as

$$\frac{100}{s^2 + 20s + 100}$$

The system is

(A) An over damped system
(B) An under damped system
(C) A critically damped system
(D) An unstable system

Statement for Linked Answer Question 27 and 28.

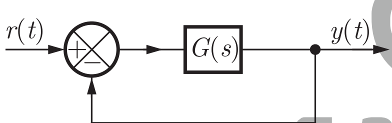
The state space equation of a system is described by $\dot{\boldsymbol{X}} = A\boldsymbol{X} + B\boldsymbol{u}, \boldsymbol{Y} = C\boldsymbol{X}$ where $\boldsymbol{X}$ is state vector, $\boldsymbol{u}$ is input, $\boldsymbol{Y}$ is output and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0]$$

MCQ 6.27 The transfer function G(s) of this system will be

(A) $\frac{s}{(s+2)}$
(B) $\frac{s+1}{s(s-2)}$
(C) $\frac{s}{(s-2)}$
(D) $\frac{1}{s(s+2)}$

MCQ 6.28 A unity feedback is provided to the above system $G(s)$ to make it a closed loop system as shown in figure.

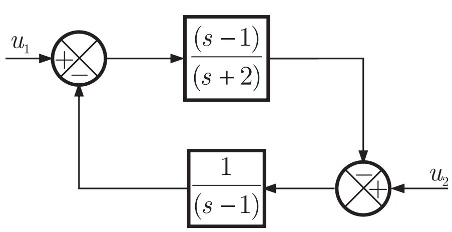


For a unit step input $r(t)$, the steady state error in the input will be

(A) 0
(B) 1
(C) 2
(D) ∞

YEAR 2007 ONE MARK

MCQ 6.29 The system shown in the figure is



(A) Stable
(B) Unstable
(C) Conditionally stable
(D) Stable for input u_1, but unstable for input u_2

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YEAR 2007 **TWO MARKS**

MCQ 6.30 If $x = \text{Re}[G(j\omega)]$, and $y = \text{Im}[G(j\omega)]$ then for $\omega \to 0^+$, the Nyquist plot for $G(s) = 1/s(s+1)(s+2)$ is

(A) $x = 0$

(B) $x = -3/4$

(C) $x = y - 1/6$

(D) $x = y/\sqrt{3}$

MCQ 6.31 The system $900/s(s+1)(s+9)$ is to be such that its gain-crossover frequency becomes same as its uncompensated phase crossover frequency and provides a 45° phase margin. To achieve this, one may use

(A) a lag compensator that provides an attenuation of 20 dB and a phase lag of 45° at the frequency of $3\sqrt{3}$ rad/s

(B) a lead compensator that provides an amplification of 20 dB and a phase lead of 45° at the frequency of 3 rad/s

(C) a lag-lead compensator that provides an amplification of 20 dB and a phase lag of 45° at the frequency of $\sqrt{3}$ rad/s

(D) a lag-lead compensator that provides an attenuation of 20 dB and phase lead of 45° at the frequency of 3 rad/s

MCQ 6.32 If the loop gain K of a negative feed back system having a loop transfer function $K(s+3)/(s+8)^2$ is to be adjusted to induce a sustained oscillation then

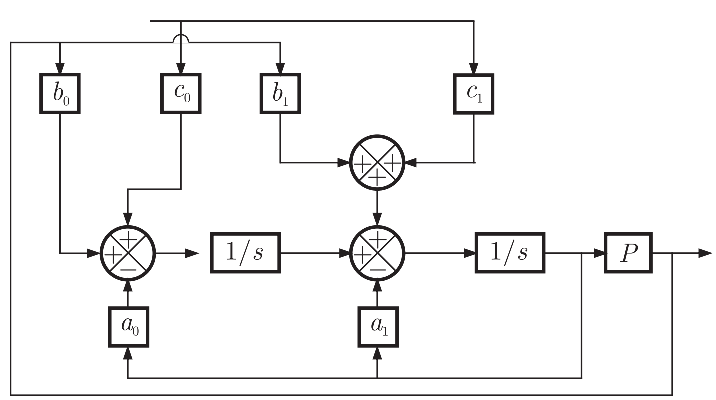
(A) The frequency of this oscillation must be $4\sqrt{3}$ rad/s

(B) The frequency of this oscillation must be 4 rad/s

(C) The frequency of this oscillation must be 4 or $4\sqrt{3}$ rad/s

(D) Such a K does not exist

MCQ 6.33 The system shown in figure below



can be reduced to the form

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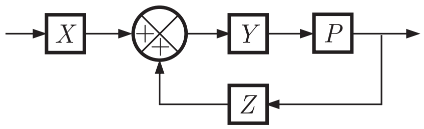


with

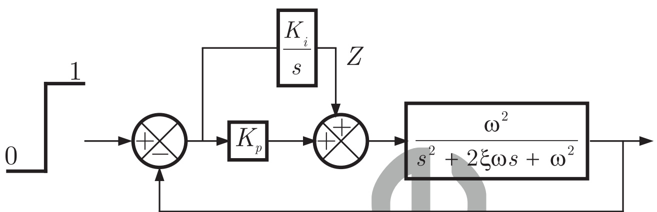
(A) $X = c_0 s + c_1, Y = 1/(s^2 + a_0 s + a_1), Z = b_0 s + b_1$

(B) $X = 1, Y = (c_0 s + c_1)/(s^2 + a_0 s + a_1), Z = b_0 s + b_1$

(C) $X = c_1 s + c_0, Y = (b_1 s + b_0)/(s^2 + a_1 s + a_0), Z = 1$

(D) $X = c_1 s + c_0, Y = 1/(s^2 + a_1 s + a), Z = b_1 s + b_0$

MCQ 6.34 Consider the feedback system shown below which is subjected to a unit step input. The system is stable and has following parameters $K_p = 4, K_i = 10, \omega = 500$ and $\xi = 0.7$. The steady state value of Z is

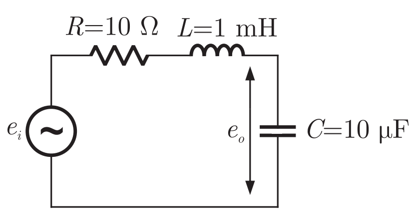


(A) 1 (B) 0.25

(C) 0.1 (D) 0

Data for Q.35 and Q.36 are given below. Solve the problems and choose the correct answers.

R-L-C circuit shown in figure



MCQ 6.35 For a step-input e_i, the overshoot in the output e_0 will be

(A) 0, since the system is not under damped

(B) 5 %

(C) 16 %

(D) 48 %

MCQ 6.36 If the above step response is to be observed on a non-storage CRO, then it would be best have the e_i as a

(A) Step function

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(B) Square wave of 50 Hz

(C) Square wave of 300 Hz

(D) Square wave of 2.0 KHz

YEAR 2006 ONE MARK

MCQ 6.37 For a system with the transfer function

$$H(s) = \frac{3(s-2)}{4s^2 - 2s + 1},$$

the matrix A in the state space form $\dot{\boldsymbol{X}} = A\boldsymbol{X} + B\boldsymbol{u}$ is equal to

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & -4 \end{bmatrix}$

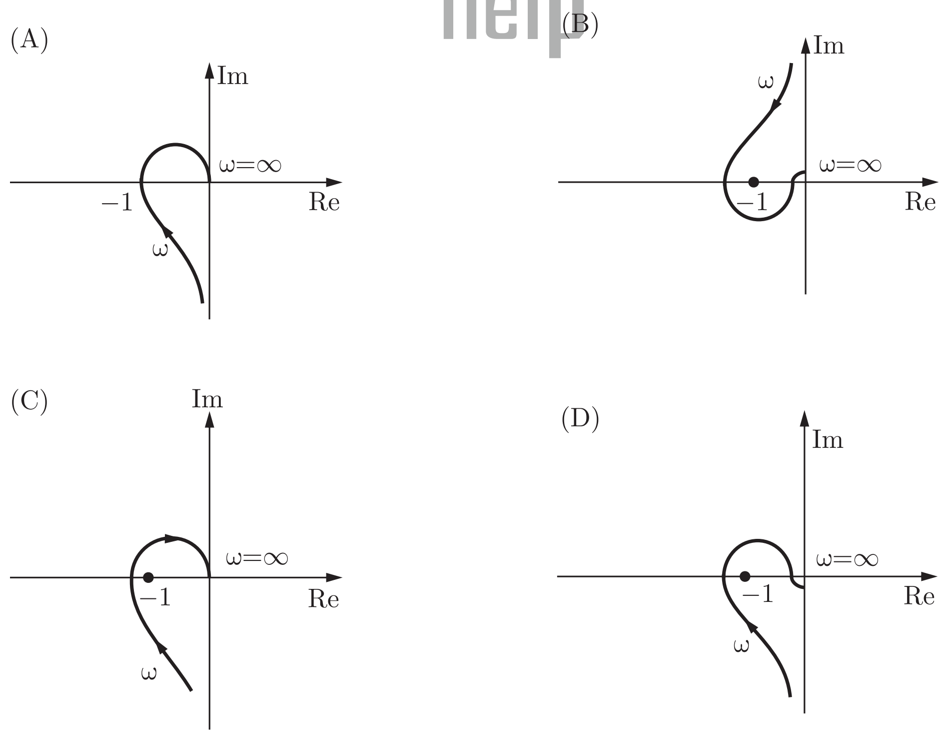
(B) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

YEAR 2006 TWO MARKS

MCQ 6.38 Consider the following Nyquist plots of loop transfer functions over $\omega = 0$ to $\omega = \infty$. Which of these plots represent a stable closed loop system ?



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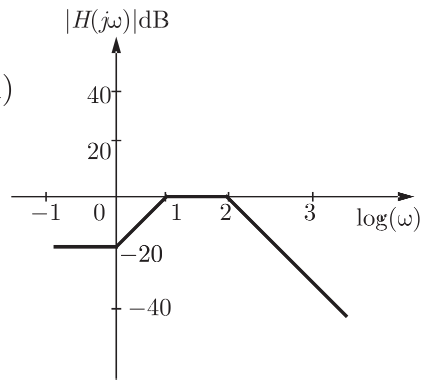
(A) (1) only (B) all, except (1)

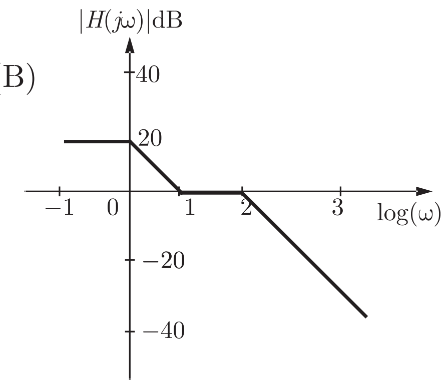
(C) all, except (3) (D) (1) and (2) only

MCQ 6.39 The Bode magnitude plot $H(j\omega) = \dfrac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$ is

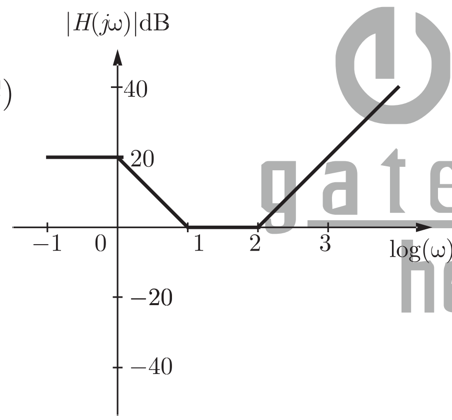
(A)



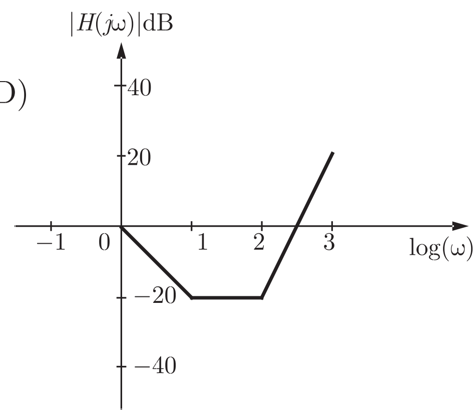
(B)



(C)

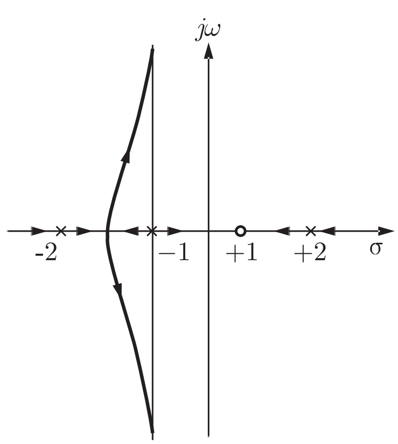


(D)

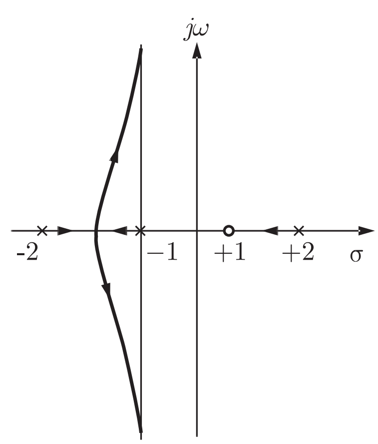


MCQ 6.40 A closed-loop system has the characteristic function $(s^2-4)(s+1)+K(s-1)=0$. Its root locus plot against K is

(A)



(B)



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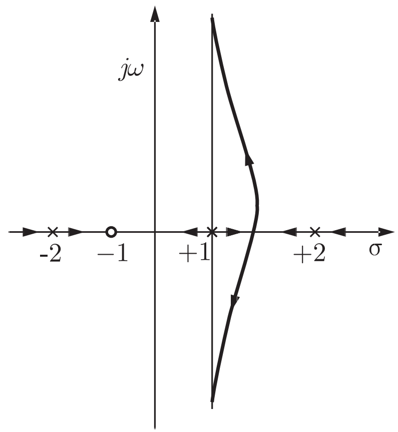
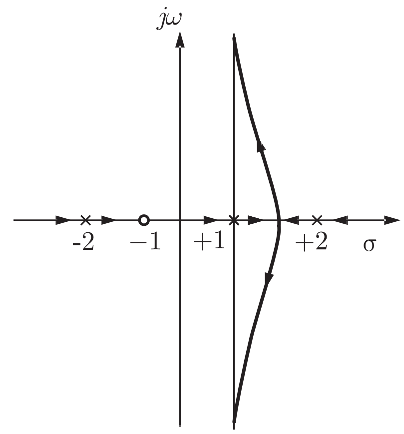
(C) 

(D) 

YEAR 2005 **ONE MARK**

MCQ 6.41 A system with zero initial conditions has the closed loop transfer function.

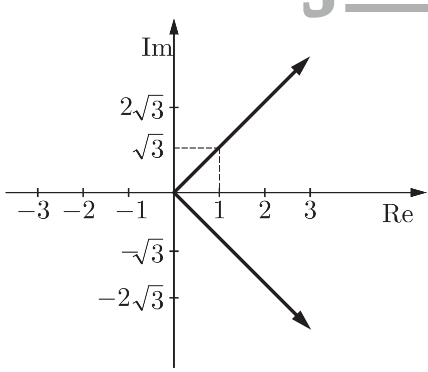
$$T(s) = \frac{s^2+4}{(s+1)(s+4)}$$

The system output is zero at the frequency

(A) 0.5 rad/sec (B) 1 rad/sec

(C) 2 rad/sec (D) 4 rad/sec

MCQ 6.42 Figure shows the root locus plot (location of poles not given) of a third order system whose open loop transfer function is



(A) $\frac{K}{s^3}$ (B) $\frac{K}{s^2(s+1)}$

(C) $\frac{K}{s(s^2+1)}$ (D) $\frac{K}{s(s^2-1)}$

MCQ 6.43 The gain margin of a unity feed back control system with the open loop transfer function $G(s) = \frac{(s+1)}{s^2}$ is

(A) 0 (B) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}$ (D) ∞

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YEAR 2005 **TWO MARKS**

MCQ 6.44 A unity feedback system, having an open loop gain

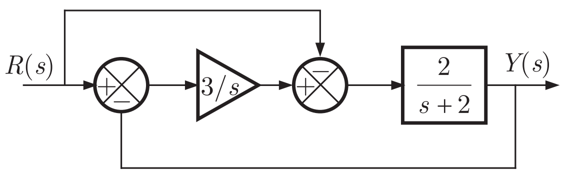
$$G(s)H(s) = \frac{K(1-s)}{(1+s)},$$

becomes stable when

(A) $|K| > 1$ (B) $K > 1$

(C) $|K| < 1$ (D) $K < -1$

MCQ 6.45 When subject to a unit step input, the closed loop control system shown in the figure will have a steady state error of



(A) -1.0 (B) -0.5

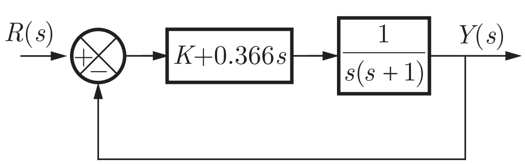
(C) 0 (D) 0.5

MCQ 6.46 In the $G(s)H(s)$-plane, the Nyquist plot of the loop transfer function $G(s)H(s) = \frac{\pi e^{-0.25s}}{s}$ passes through the negative real axis at the point

(A) $(-0.25, j0)$ (B) $(-0.5, j0)$

(C) 0 (D) 0.5

MCQ 6.47 If the compensated system shown in the figure has a phase margin of 60° at the crossover frequency of 1 rad/sec, then value of the gain K is



(A) 0.366 (B) 0.732

(C) 1.366 (D) 2.738

Data for Q.48 and Q.49 are given below. Solve the problem and choose the correct answer.

A state variable system $\dot{\boldsymbol{X}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \boldsymbol{X}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \boldsymbol{u}(t)$ with the initial condition $\boldsymbol{X}(0) = [-1,\ 3]^T$ and the unit step input $u(t)$ has

MCQ 6.48 The state transition matrix

(A) $\begin{bmatrix} 1 & \frac{1}{3}(1-e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$

(B) $\begin{bmatrix} 1 & \frac{1}{3}(e^{-t}-e^{-3t}) \\ 0 & e^{-t} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & \frac{1}{3}(e^{3-t}-e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$

(D) $\begin{bmatrix} 1 & (1-e^{-t}) \\ 0 & e^{-t} \end{bmatrix}$

MCQ 6.49 The state transition equation

(A) $\boldsymbol{X}(t) = \begin{bmatrix} t-e^{-t} \\ e^{-t} \end{bmatrix}$

(B) $\boldsymbol{X}(t) = \begin{bmatrix} 1-e^{-t} \\ 3e^{-3t} \end{bmatrix}$

(C) $\boldsymbol{X}(t) = \begin{bmatrix} t-e^{3t} \\ 3e^{-3t} \end{bmatrix}$

(D) $\boldsymbol{X}(t) = \begin{bmatrix} t-e^{-3t} \\ e^{-t} \end{bmatrix}$

YEAR 2004 ONE MARK

MCQ 6.50 The Nyquist plot of loop transfer function $G(s)H(s)$ of a closed loop control system passes through the point $(-1, j0)$ in the $G(s)H(s)$plane. The phase margin of the system is

(A) 0°

(B) 45°

(C) 90°

(D) 180°

MCQ 6.51 Consider the function,

$$F(s) = \frac{5}{s(s^2+3s+2)}$$

where $F(s)$ is the Laplace transform of the of the function $f(t)$. The initial value of $f(t)$ is equal to

(A) 5

(B) $\frac{5}{2}$

(C) $\frac{5}{3}$

(D) 0

MCQ 6.52 For a tachometer, if $\theta(t)$ is the rotor displacement in radians, $e(t)$ is the output voltage and K_t is the tachometer constant in V/rad/sec, then the transfer function, $\frac{E(s)}{Q(s)}$ will be

(A) $K_t s^2$

(B) K_t/s

(C) $K_t s$

(D) K_t

YEAR 2004 TWO MARKS

MCQ 6.53 For the equation, $s^3-4s^2+s+6=0$ the number of roots in the left half of s-plane will be

(A) Zero

(B) One

(C) Two

(D) Three

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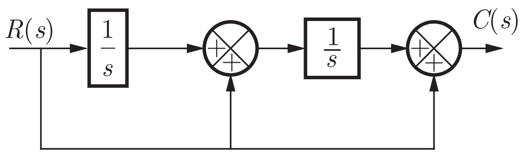
MCQ 6.54 For the block diagram shown, the transfer function $\frac{C(s)}{R(s)}$ is equal to



(A) $\frac{s^2+1}{s^2}$ (B) $\frac{s^2+s+1}{s^2}$

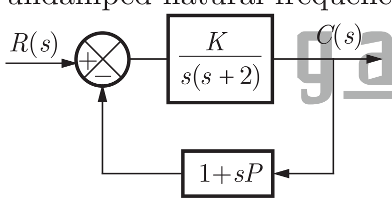
(C) $\frac{s^2+s+1}{s}$ (D) $\frac{1}{s^2+s+1}$

MCQ 6.55 The state variable description of a linear autonomous system is, $\dot{\boldsymbol{X}} = A\boldsymbol{X}$ where $\boldsymbol{X}$ is the two dimensional state vector and A is the system matrix given by $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$. The roots of the characteristic equation are

(A) -2 and $+2$ (B) $-j2$ and $+j2$

(C) -2 and -2 (D) $+2$ and $+2$

MCQ 6.56 The block diagram of a closed loop control system is given by figure. The values of K and P such that the system has a damping ratio of 0.7 and an undamped natural frequency ω_n of 5 rad/sec, are respectively equal to



(A) 20 and 0.3 (B) 20 and 0.2

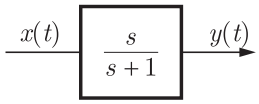
(C) 25 and 0.3 (D) 25 and 0.2

MCQ 6.57 The unit impulse response of a second order under-damped system starting from rest is given by $c(t) = 12.5e^{-6t}\sin 8t,\ \ t \geq 0$. The steady-state value of the unit step response of the system is equal to

(A) 0 (B) 0.25

(C) 0.5 (D) 1.0

MCQ 6.58 In the system shown in figure, the input $x(t) = \sin t$. In the steady-state, the response $y(t)$ will be



(A) $\frac{1}{\sqrt{2}}\sin(t-45^\circ)$ (B) $\frac{1}{\sqrt{2}}\sin(t+45^\circ)$

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(C) $\sin(t-45^\circ)$ (D) $\sin(t+45^\circ)$

MCQ 6.59 The open loop transfer function of a unity feedback control system is given as

$$G(s) = \frac{as+1}{s^2}.$$

The value of 'a' to give a phase margin of 45° is equal to

(A) 0.141 (B) 0.441

(C) 0.841 (D) 1.141

YEAR 2003 ONE MARK

MCQ 6.60 A control system is defined by the following mathematical relationship

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 12(1-e^{-2t})$$

The response of the system as $t \to \infty$ is

(A) $x = 6$ (B) $x = 2$

(C) $x = 2.4$ (D) $x = -2$

MCQ 6.61 A lead compensator used for a closed loop controller has the following transfer function

$$\frac{K(1+\frac{s}{a})}{(1+\frac{s}{b})}$$

For such a lead compensator

(A) $a < b$ (B) $b < a$

(C) $a > Kb$ (D) $a < Kb$

MCQ 6.62 A second order system starts with an initial condition of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ without any external input. The state transition matrix for the system is given by $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}$. The state of the system at the end of 1 second is given by

(A) $\begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$ (B) $\begin{bmatrix} 0.135 \\ 0.368 \end{bmatrix}$

(C) $\begin{bmatrix} 0.271 \\ 0.736 \end{bmatrix}$ (D) $\begin{bmatrix} 0.135 \\ 1.100 \end{bmatrix}$

YEAR 2003 TWO MARKS

MCQ 6.63 A control system with certain excitation is governed by the following mathematical equation

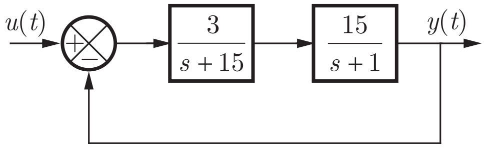
$$\frac{d^2x}{dt^2} + \frac{1}{2}\frac{dx}{dt} + \frac{1}{18}x = 10 + 5e^{-4t} + 2e^{-5t}$$

The natural time constant of the response of the system are

(A) 2 sec and 5 sec (B) 3 sec and 6 sec

(C) 4 sec and 5 sec (D) 1/3 sec and 1/6 sec

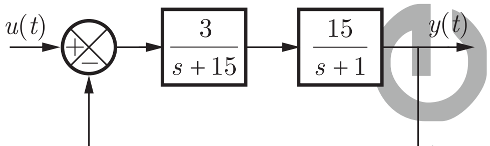
MCQ 6.64 The block diagram shown in figure gives a unity feedback closed loop control system. The steady state error in the response of the above system to unit step input is



(A) 25% (B) 0.75 %

(C) 6% (D) 33%

MCQ 6.65 The roots of the closed loop characteristic equation of the system shown above (Q-5.55)



(A) -1 and -15 (B) 6 and 10

(C) -4 and -15 (D) -6 and -10

MCQ 6.66 The following equation defines a separately excited dc motor in the form of a differential equation

$$\frac{d^2\omega}{dt} + \frac{B}{J}\frac{d\omega}{dt} + \frac{K^2}{LJ}\omega = \frac{K}{LJ}V_a$$

The above equation may be organized in the state-space form as follows

$$\begin{bmatrix} \frac{d^2\omega}{dt^2} \\ \frac{d\omega}{dt} \end{bmatrix} = P\begin{bmatrix} \frac{d\omega}{dt} \\ \omega \end{bmatrix} + QV_a$$

Where the P matrix is given by

(A) $\begin{bmatrix} -\frac{B}{J} & -\frac{K^2}{LJ} \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} -\frac{K^2}{LJ} & -\frac{B}{J} \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 \\ -\frac{K^2}{LJ} & -\frac{B}{J} \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ -\frac{B}{J} & -\frac{K^2}{LJ} \end{bmatrix}$

MCQ 6.67 The loop gain GH of a closed loop system is given by the following expression

$$\frac{K}{s(s+2)(s+4)}$$

The value of K for which the system just becomes unstable is

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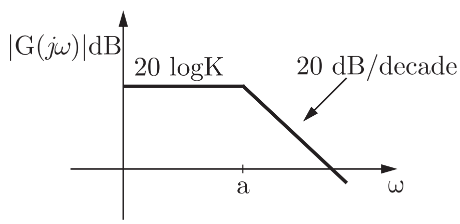
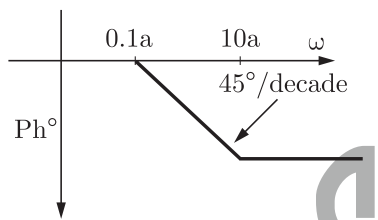
(A) $K = 6$ (B) $K = 8$
(C) $K = 48$ (D) $K = 96$

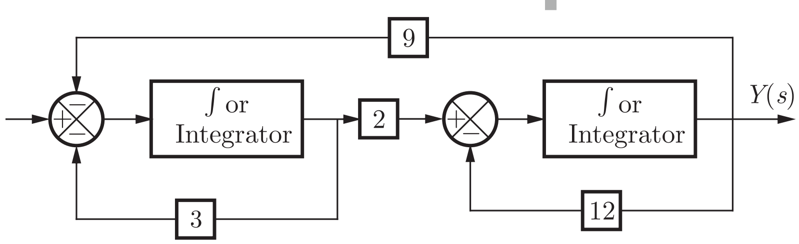
MCQ 6.68 The asymptotic Bode plot of the transfer function $K/[1 + (s/a)]$ is given in figure. The error in phase angle and dB gain at a frequency of $\omega = 0.5a$ are respectively





(A) 4.9°, 0.97 dB (B) 5.7°, 3 dB
(C) 4.9°, 3 dB (D) 5.7°, 0.97 dB

MCQ 6.69 The block diagram of a control system is shown in figure. The transfer function $G(s) = Y(s)/U(s)$ of the system is



(A) $\dfrac{1}{18\left(1+\frac{s}{12}\right)\left(1+\frac{s}{3}\right)}$ (B) $\dfrac{1}{27\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$

(C) $\dfrac{1}{27\left(1+\frac{s}{12}\right)\left(1+\frac{s}{9}\right)}$ (D) $\dfrac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{3}\right)}$

YEAR 2002 ONE MARK

MCQ 6.70 The state transition matrix for the system $\dot{\boldsymbol{X}} = A\boldsymbol{X}$ with initial state $\boldsymbol{X}(0)$ is

(A) $(sI - A)^{-1}$

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(B) $e^{At}\boldsymbol{X}(0)$

(C) Laplace inverse of $[(sI - A)^{-1}]$

(D) Laplace inverse of $[(sI - A)^{-1}\boldsymbol{X}(0)]$

YEAR 2002 TWO MARKS

MCQ 6.71 For the system $\dot{\boldsymbol{X}} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}\boldsymbol{X} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\boldsymbol{u}$, which of the following statements is true ?

(A) The system is controllable but unstable

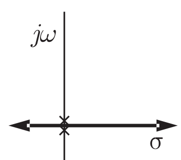
(B) The system is uncontrollable and unstable

(C) The system is controllable and stable

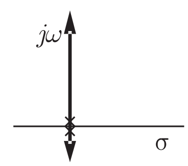
(D) The system is uncontrollable and stable

MCQ 6.72 A unity feedback system has an open loop transfer function, $G(s) = \frac{K}{s^2}$. The root locus plot is

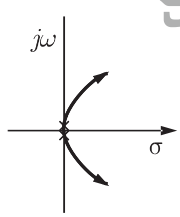
(A)



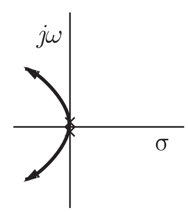
(B)



(C)



(D)



MCQ 6.73 The transfer function of the system described by

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} = \frac{du}{dt} + 2u$$

with u as input and y as output is

(A) $\frac{(s+2)}{(s^2+s)}$ (B) $\frac{(s+1)}{(s^2+s)}$

(C) $\frac{2}{(s^2+s)}$ (D) $\frac{2s}{(s^2+s)}$

MCQ 6.74 For the system

$$\dot{\boldsymbol{X}} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\boldsymbol{X} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\boldsymbol{u};\ \boldsymbol{Y} = \begin{bmatrix} 4 & 0 \end{bmatrix}\boldsymbol{X},$$

with u as unit impulse and with zero initial state, the output y, becomes

(A) $2e^{2t}$ (B) $4e^{2t}$

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(C) $2e^{4t}$ (D) $4e^{4t}$

MCQ 6.75 The eigen values of the system represented by

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \text{ are}$$

(A) 0, 0, 0, 0 (B) 1, 1, 1, 1

(C) 0, 0, 0, -1 (D) 1, 0, 0, 0

MCQ 6.76 *A single input single output system with y as output and u as input, is described by

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 5\frac{du}{dt} - 3u$$

for an input $u(t)$ with zero initial conditions the above system produces the same output as with no input and with initial conditions

$$\frac{dy(0^-)}{dt} = -4,\ y(0^-) = 1$$

input $u(t)$ is

(A) $\frac{1}{5}\delta(t) - \frac{7}{25}e^{(3/5)t}u(t)$ (B) $\frac{1}{5}\delta(t) - \frac{7}{25}e^{-3t}u(t)$

(C) $-\frac{7}{25}e^{-(3/5)t}u(t)$ (D) None of these

MCQ 6.77 *A system is described by the following differential equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = u(t)\,e^{-t}$$

the state variables are given as $x_1 = y$ and $x_2 = \left(\frac{dy}{dt} - y\right)e^t$, the state varibale representation of the system is

(A) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & e^{-t} \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(t)$

(B) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(t)$

(C) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & e^{-t} \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(t)$

(D) none of these

Common Data Question Q.78-80*.

The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{2(s+\alpha)}{s(s+2)(s+10)}$$

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MCQ 6.78 Angles of asymptotes are

(A) $60°, 120°, 300°$
(B) $60°, 180°, 300°$
(C) $90°, 270°, 360°$
(D) $90°, 180°, 270°$

MCQ 6.79 Intercepts of asymptotes at the real axis is

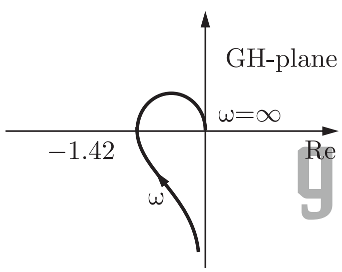
(A) -6
(B) $-\frac{10}{3}$
(C) -4
(D) -8

MCQ 6.80 Break away points are

(A) $-1.056, -3.471$
(B) $-2.112, -6.9433$
(C) $-1.056, -6.9433$
(D) $1.056, -6.9433$

YEAR 2001 ONE MARK

MCQ 6.81 The polar plot of a type-1, 3-pole, open-loop system is shown in Figure The closed-loop system is



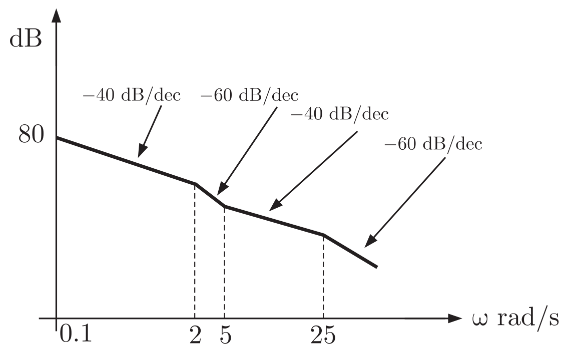
(A) always stable
(B) marginally stable
(C) unstable with one pole on the right half s-plane
(D) unstable with two poles on the right half s-plane.

MCQ 6.82 Given the homogeneous state-space equation $\dot{x} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} x$ the steady state value of $x_{ss} = \lim_{t \to \infty} x(t)$, given the initial state value of $x(0) = \begin{bmatrix} 10 & -10 \end{bmatrix}^T$ is

(A) $x_{ss} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
(B) $x_{ss} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$
(C) $x_{ss} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$
(D) $x_{ss} = \begin{bmatrix} \infty \\ \infty \end{bmatrix}$

YEAR 2001 TWO MARKS

MCQ 6.83 The asymptotic approximation of the log-magnitude versus frequency plot of a minimum phase system with real poles and one zero is shown in Figure. Its transfer functions is



(A) $\dfrac{20(s+5)}{s(s+2)(s+25)}$

(B) $\dfrac{10(s+5)}{(s+2)^2(s+25)}$

(C) $\dfrac{20(s+5)}{s^2(s+2)(s+25)}$

(D) $\dfrac{50(s+5)}{s^2(s+2)(s+25)}$

Common Data Question Q.84-87*.

A unity feedback system has an open-loop transfer function of

$$G(s) = \frac{10000}{s(s+10)^2}$$

MCQ 6.84 Determine the magnitude of $G(j\omega)$ in dB at an angular frequency of $\omega = 20$ rad/sec.

(A) 1 dB

(B) 0 dB

(C) -2 dB

(D) 10 dB

MCQ 6.85 The phase margin in degrees is

(A) 90°

(B) 36.86°

(C) -36.86°

(D) -90°

MCQ 6.86 The gain margin in dB is

(A) 13.97 dB

(B) 6.02 dB

(C) -13.97 dB

(D) None of these

MCQ 6.87 The system is

(A) Stable

(B) Un-stable

(C) Marginally stable

(D) can not determined

MCQ 6.88 *For the given characteristic equation

$$s^3 + s^2 + Ks + K = 0$$

The root locus of the system as K varies from zero to infinity is

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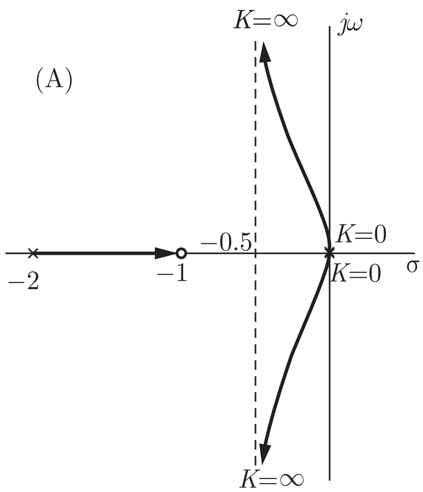
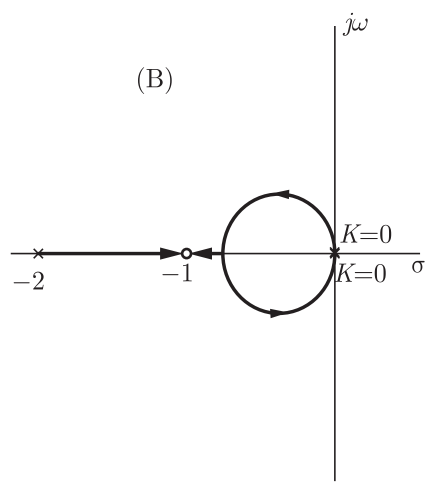
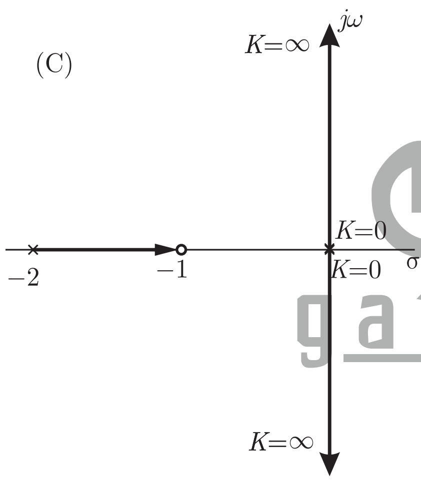
(A)



(B)



(C)



(D) none of these

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SOLUTION

SOL 6.1 Option (D) is correct.

General form of state equations are given as

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u$$
$$\dot{y} = \boldsymbol{C}x + \boldsymbol{D}u$$

For the given problem

$$\boldsymbol{A} = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{AB} = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ a_2 \\ 0 \end{bmatrix}$$

$$\boldsymbol{A}^2\boldsymbol{B} = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ a_2 a_3 & 0 & 0 \\ 0 & a_3 a_1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 a_2 \\ 0 \\ 0 \end{bmatrix}$$

For controllability it is necessary that following matrix has a tank of $n = 3$.

$$\boldsymbol{U} = [\boldsymbol{B} : \boldsymbol{AB} : \boldsymbol{A}^2\boldsymbol{B}] = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

So,
$$a_2 \neq 0$$
$$a_1 a_2 \neq 0 \Rightarrow \ a_1 \neq 0$$

a_3 may be zero or not.

SOL 6.2 Option (A) is correct.

$$Y(s) = \frac{K(s+1)}{s^3 + as^2 + 2s + 1}[R(s) - Y(s)]$$

$$Y(s)\left[1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1}\right] = \frac{K(s+1)}{s^3 + as^2 + 2s + 1}R(s)$$

$$Y(s)\,[s^3 + as^2 + s(2+k) + (1+k)] = K(s+1)\,R(s)$$

Transfer Function, $$H(s) = \frac{Y(s)}{R(s)} = \frac{K(s+1)}{s^3 + as^2 + s(2+k) + (1+k)}$$

Routh Table :

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s^3	1	$2+K$
s^2	a	$1+K$
s^1	$\frac{a(2+K)-(1+K)}{a}$	0

For oscillation,

$$\frac{a(2+K)-(1+K)}{a} = 0$$

$$a = \frac{K+1}{K+2}$$

Auxiliary equation

$$A(s) = as^2 + (k+1) = 0$$

$$s^2 = -\frac{k+1}{a}$$

$$s^2 = \frac{-k+1}{(k+1)}(k+2) = -(k+2)$$

$$s = j\sqrt{k+2}$$

$$j\omega = j\sqrt{k+2}$$

$$\omega = \sqrt{k+2} = 2$$

(Oscillation frequency)

$$k = 2$$

and $$a = \frac{2+1}{2+2} = \frac{3}{4} = 0.75$$

SOL 6.3 Option (A) is correct.

$$G_C(s) = \frac{s+a}{s+b} = \frac{j\omega+a}{j\omega+b}$$

Phase lead angle, $$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\omega}{a}-\frac{\omega}{b}}{1+\frac{\omega^2}{ab}}\right) = \tan^{-1}\left(\frac{\omega(b-a)}{ab+\omega^2}\right)$$

For phase-lead compensation $\phi > 0$

$$b - a > 0$$

$$b > a$$

Note: For phase lead compensator zero is nearer to the origin as compared to pole, so option (C) can not be true.

SOL 6.4 Option (A) is correct.

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

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$$\frac{d\phi}{d\omega} = \frac{1/a}{1+\left(\frac{\omega}{a}\right)^2} - \frac{1/b}{1+\left(\frac{\omega}{b}\right)^2} = 0$$

$$\frac{1}{a} + \frac{\omega^2}{ab^2} = \frac{1}{b} + \frac{1}{b}\frac{\omega^2}{a^2}$$

$$\frac{1}{a} - \frac{1}{b} = \frac{\omega^2}{ab}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\omega = \sqrt{ab} = \sqrt{1 \times 2} = \sqrt{2} \text{ rad/sec}$$

SOL 6.5 Option (A) is correct.

Gain margin is simply equal to the gain at phase cross over frequency (ω_p). Phase cross over frequency is the frequency at which phase angle is equal to $-180°$.

From the table we can see that $\angle G(j\omega_p) = -180°$, at which gain is 0.5.

$$GM = 20\log_{10}\left(\frac{1}{|G(j\omega_p)|}\right) = 20\log\left(\frac{1}{0.5}\right) = 6 \text{ dB}$$

Phase Margin is equal to 180° plus the phase angle ϕ_g at the gain cross over frequency (ω_g). Gain cross over frequency is the frequency at which gain is unity.

From the table it is clear that $|G(j\omega_g)| = 1$, at which phase angle is $-150°$

$$\phi_{PM} = 180° + \angle G(j\omega_g) = 180 - 150 = 30°$$

SOL 6.6 Option (A) is correct.

We know that steady state error is given by

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

where $R(s) \rightarrow$ input

$G(s) \rightarrow$ open loop transfer function

For unit step input

$$R(s) = \frac{1}{s}$$

So $$e_{ss} = \lim_{s \to 0} \frac{s\left(\frac{1}{s}\right)}{1+G(s)} = 0.1$$

$$1 + G(0) = 10$$

$$G(0) = 9$$

Given input $$r(t) = 10[\mu(t) - \mu(t-1)]$$

or $$R(s) = 10\left[\frac{1}{s} - \frac{1}{s}e^{-s}\right] = 10\left[\frac{1-e^{-s}}{s}\right]$$

So steady state error

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$$e'_{ss} = \lim_{s \to 0} \frac{s \times 10 \frac{(1-e^{-s})}{s}}{1+G(s)} = \frac{10(1-e^{0})}{1+9} = 0$$

SOL 6.7 Option (B) is correct.

Transfer function having at least one zero or pole in RHS of s-plane is called non-minimum phase transfer function.

$$G(s) = \frac{s-1}{(s+2)(s+3)}$$

- In the given transfer function one zero is located at $s = 1$ (RHS), so this is a non-minimum phase system.
- Poles $-2, -3$, are in left side of the complex plane, So the system is stable

SOL 6.8 Option (A) is correct.

$$G(s) = \frac{K\left(s+\frac{2}{3}\right)}{s^2(s+2)}$$

Steps for plotting the root-locus

(1) Root loci starts at $s = 0, s = 0$ and $s = -2$

(2) $n > m$, therefore, number of branches of root locus $b = 3$

(3) Angle of asymptotes is given by

$$\frac{(2q+1)180^\circ}{n-m}, \; q = 0, 1$$

(I) $\dfrac{(2 \times 0 + 1)180^\circ}{(3-1)} = 90^\circ$

(II) $\dfrac{(2 \times 1 + 1)180^\circ}{(3-1)} = 270^\circ$

(4) The two asymptotes intersect on real axis at centroid

$$x = \frac{\sum \text{Poles} - \sum \text{Zeroes}}{n-m} = \frac{-2-\left(-\frac{2}{3}\right)}{3-1} = -\frac{2}{3}$$

(5) Between two open-loop poles $s = 0$ and $s = -2$ there exist a break away point.

$$K = -\frac{s^2(s+2)}{\left(s+\frac{2}{3}\right)}$$

$$\frac{dK}{ds} = 0$$

$$s = 0$$

Root locus is shown in the figure

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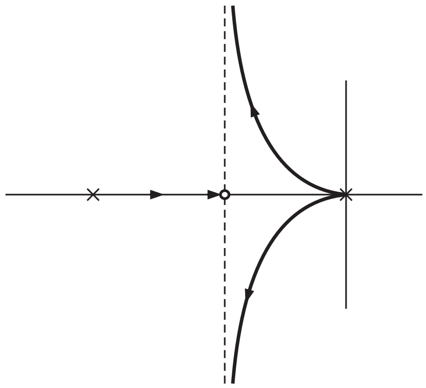
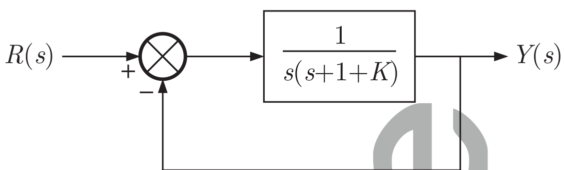


Three roots with nearly equal parts exist on the left half of s-plane.

SOL 6.9 Option (A) is correct.

The system may be reduced as shown below



$$\frac{Y(s)}{R(s)} = \frac{\frac{1}{s(s+1+K)}}{1+\frac{1}{s(s+1+K)}} = \frac{1}{s^2+s(1+K)+1}$$

This is a second order system transfer function, characteristic equation is

$$s^2 + s(1+K) + 1 = 0$$

Comparing with standard form

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

We get $\quad \xi = \frac{1+K}{2}$

Peak overshoot

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

So the Peak overshoot is effected by K.

SOL 6.10 Option (A) is correct.

Given $\quad G(s) = \frac{1}{s(s+1)(s+2)}$

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

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$$|G(j\omega)| = \frac{1}{\omega\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(\omega/2)$$

In nyquist plot

For $\omega = 0$, $|G(j\omega)| = \infty$

$$\angle G(j\omega) = -90^\circ$$

For $\omega = \infty$, $|G(j\omega)| = 0$

$$\angle G(j\omega) = -90^\circ - 90^\circ - 90^\circ = -270^\circ$$

Intersection at real axis

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)} = \frac{1}{j\omega(-\omega^2+j3\omega+2)}$$

$$= \frac{1}{-3\omega^2+j\omega(2-\omega^2)} \times \frac{-3\omega^2-j\omega(2-\omega^2)}{-3\omega^2-j\omega(2-\omega^2)}$$

$$= \frac{-3\omega^2-j\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2}$$

$$= \frac{-3\omega^2}{9\omega^4+\omega^2(2-\omega^2)^2} - \frac{j\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2}$$

At real axis

$$\text{Im}[G(j\omega)] = 0$$

So, $\frac{\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)} = 0$

$$2-\omega^2 = 0 \Rightarrow \omega = \sqrt{2} \text{ rad/sec}$$

At $\omega = \sqrt{2}$ rad/sec, magnitude response is

$$|G(j\omega)|_{at\ \omega=\sqrt{2}} = \frac{1}{\sqrt{2}\sqrt{2+1}\sqrt{2+4}} = \frac{1}{6} < \frac{3}{4}$$

SOL 6.11 Option (C) is correct.

Stability :

Eigen value of the system are calculated as

$$|A-\lambda I| = 0$$

$$A - \lambda I = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} -1-\lambda & 2 \\ 0 & 2-\lambda \end{bmatrix}$$

$$|A-\lambda I| = (-1-\lambda)(2-\lambda) - 2\times 0 = 0$$

$\Rightarrow$ $\lambda_1, \lambda_2 = -1, 2$

Since eigen values of the system are of opposite signs, so it is unstable

Controllability :

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$[B{:}\,AB] = \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$$

$$[B{:}\,AB] \neq 0$$

So it is controllable.

SOL 6.12 Option (C) is correct.

Given characteristic equation

$$s(s+1)(s+3) + K(s+2) = 0; \qquad K > 0$$

$$s(s^2 + 4s + 3) + K(s+2) = 0$$

$$s^3 + 4s^2 + (3+K)s + 2K = 0$$

From Routh's tabulation method

s^3	1	$3+K$
s^2	4	$2K$
s^1	$\dfrac{4(3+K) - 2K(1)}{4} = \dfrac{12+2K}{4} > 0$	
s^0	$2K$	

There is no sign change in the first column of routh table, so no root is lying in right half of s-plane.

For plotting root locus, the equation can be written as

$$1 + \frac{K(s+2)}{s(s+1)(s+3)} = 0$$

Open loop transfer function

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)}$$

Root locus is obtained in following steps:

1. No. of poles $n = 3$, at $s = 0, s = -1$ and $s = -3$
2. No. of Zeroes $m = 1$, at $s = -2$
3. The root locus on real axis lies between $s = 0$ and $s = -1$, between $s = -3$ and $s = -2$.
4. Breakaway point lies between open loop poles of the system. Here breakaway point lies in the range $-1 < \text{Re}[s] < 0$.
5. Asymptotes meet on real axis at a point C, given by

$$C = \frac{\sum \text{real part of poles} - \sum \text{real parts of zeroes}}{n-m}$$

$$= \frac{(0-1-3) - (-2)}{3-1}$$

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$$= -1$$

As no. of poles is 3, so two root loci branches terminates at infinity along asymptotes $\text{Re}(s) = -1$

SOL 6.13 Option (D) is correct.

Overall gain of the system is written as

$$G = G_1 G_2 \frac{1}{G_3}$$

We know that for a quantity that is product of two or more quantities total percentage error is some of the percentage error in each quantity. so error in overall gain G is

$$\triangle G = \varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3}$$

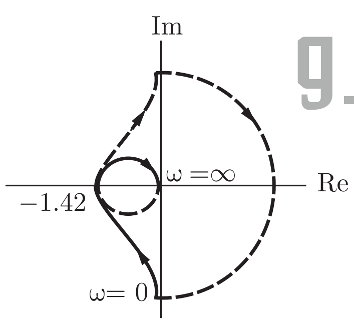
SOL 6.14 Option (D) is correct.

From Nyquist stability criteria, no. of closed loop poles in right half of s -plane is given as

$$Z = P - N$$

$P \rightarrow$ No. of open loop poles in right half s-plane

$N \rightarrow$ No. of encirclement of $(-1, j0)$



Here $N = -2$ ($\therefore$ encirclement is in clockwise direction)

$P = 0$ ($\therefore$ system is stable)

So, $Z = 0 - (-2)$

$Z = 2$, System is unstable with 2-poles on RH of s-plane.

SOL 6.15 Option (D) is correct.

Given Routh's tabulation.

s^3	2	2
s^2	4	4
s^1	0	0

So the auxiliary equation is given by,

$$4s^2 + 4 = 0$$

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$$s^2 = -1$$
$$s = \pm j$$

From table we have characteristic equation as

$$2s^3 + 2s + 4s^2 + 4 = 0$$
$$s^3 + s + 2s^2 + 2 = 0$$
$$s(s^2+1) + 2(s^2+1) = 0$$
$$(s+2)(s^2+1) = 0$$
$$s = -2,\ s = \pm j$$

SOL 6.16 Option (B) is correct.

Since initial slope of the bode plot is -40 dB/decade, so no. of poles at origin is 2.

Transfer function can be written in following steps:

1. Slope changes from -40 dB/dec. to -60 dB/dec. at $\omega_1 = 2$ rad/sec., so at ω_1 there is a pole in the transfer function.
2. Slope changes from -60 dB/dec to -40 dB/dec at $\omega_2 = 5$ rad/sec., so at this frequency there is a zero lying in the system function.
3. The slope changes from -40 dB/dec to -60 dB/dec at $\omega_3 = 25$ rad/sec, so there is a pole in the system at this frequency.

Transfer function

$$T(s) = \frac{K(s+5)}{s^2(s+2)(s+25)}$$

Constant term can be obtained as.

$$T(j\omega)\big|_{at\ \omega = 0.1} = 80$$

So,

$$80 = 20\log\frac{K(5)}{(0.1)^2 \times 50}$$
$$K = 1000$$

therefore, the transfer function is

$$T(s) = \frac{1000(s+5)}{s^2(s+2)(s+25)}$$

SOL 6.17 Option (D) is correct.

From the figure we can see that steady state error for given system is

$$e_{ss} = 1 - 0.75 = 0.25$$

Steady state error for unity feed back system is given by

$$e_{ss} = \lim_{s\to 0}\left[\frac{sR(s)}{1+G(s)}\right]$$
$$= \lim_{s\to 0}\left[\frac{s\left(\frac{1}{s}\right)}{1+\frac{K}{(s+1)(s+2)}}\right];\ R(s) = \frac{1}{s}\ \text{(unit step input)}$$

$$= \frac{1}{1+\frac{K}{2}} = \frac{2}{2+K}$$

So, $$e_{ss} = \frac{2}{2+K} = 0.25$$

$$2 = 0.5 + 0.25K$$

$$K = \frac{1.5}{0.25} = 6$$

SOL 6.18 Option (D) is correct.

Open loop transfer function of the figure is given by,

$$G(s) = \frac{e^{-0.1s}}{s}$$

$$G(j\omega) = \frac{e^{-j0.1\omega}}{j\omega}$$

Phase cross over frequency can be calculated as,

$$\angle G(j\omega_p) = -180^\circ$$

$$\left(-0.1\omega_p \times \frac{180}{\pi}\right) - 90^\circ = -180^\circ$$

$$0.1\omega_p \times \frac{180^\circ}{\pi} = 90^\circ$$

$$0.1\omega_p = \frac{90^\circ \times \pi}{180^\circ}$$

$$\omega_p = 15.7 \text{ rad/sec}$$

So the gain margin (dB) $$= 20\log\left(\frac{1}{|G(j\omega_p)|}\right) = 20\log\left[\frac{1}{\left(\frac{1}{15.7}\right)}\right]$$

$$= 20\log 15.7 = 23.9 \text{ dB}$$

SOL 6.19 Option (C) is correct.

Given system equations

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

$$y(t) = x_1(t)$$

Taking Laplace transform on both sides of equations.

$$sX_1(s) = -3X_1(s) + X_2(s) + 2U(s)$$

$$(s+3)X_1(s) = X_2(s) + 2U(s) \quad ...(1)$$

Similarly $$sX_2(s) = -2X_2(s) + U(s)$$

$$(s+2)X_2(s) = U(s) \quad ...(2)$$

From equation (1) & (2)

$$(s+3)\,X_1(s) = \frac{U(s)}{s+2} + 2\,U(s)$$

$$X_1(s) = \frac{U(s)}{s+3}\left[\frac{1+2\,(s+2)}{s+2}\right] = U(s)\frac{(2s+5)}{(s+2)\,(s+3)}$$

From output equation,

$$Y(s) = X_1(s)$$

So,

$$Y(s) = U(s)\frac{(2s+5)}{(s+2)\,(s+3)}$$

System transfer function

$$\text{T.F} = \frac{Y(s)}{U(s)} = \frac{(2s+5)}{(s+2)\,(s+3)} = \frac{(2s+5)}{s^2+5s+6}$$

SOL 6.20 Option (B) is correct.

Given state equations in matrix form can be written as,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2\\ 1\end{bmatrix}\boldsymbol{u}(t)$$

$$\frac{d\boldsymbol{X}(t)}{dt} = A\boldsymbol{X}(t) + B\boldsymbol{u}(t)$$

State transition matrix is given by

$$\phi(t) = \mathcal{L}^{-1}[\boldsymbol{\Phi}(s)]$$

$$\boldsymbol{\Phi}(s) = (sI-A)^{-1}$$

$$(sI-A) = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}-3 & 1\\ 0 & -2\end{bmatrix}$$

$$(sI-A) = \begin{bmatrix}s+3 & -1\\ 0 & s+2\end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{(s+3)\,(s+2)}\begin{bmatrix}s+2 & 1\\ 0 & s+3\end{bmatrix}$$

$$\text{So}\ \boldsymbol{\Phi}(s) = (sI-A)^{-1} = \begin{bmatrix}\frac{1}{(s+3)} & \frac{1}{(s+3)\,(s+2)}\\ 0 & \frac{1}{(s+2)}\end{bmatrix}$$

$$\phi(t) = \mathcal{L}^{-1}[\boldsymbol{\Phi}(s)] = \begin{bmatrix}e^{-3t} & e^{-2t}-e^{-3t}\\ 0 & e^{-2t}\end{bmatrix}$$

SOL 6.21 Option (D) is correct.

Given differential equation for the function

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

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Taking Laplace on both the sides we have,

$$sY(s) + Y(s) = 1$$

$$(s+1)\,Y(s) = 1$$

$$Y(s) = \frac{1}{s+1}$$

Taking inverse Laplace of $Y(s)$

$$y(t) = e^{-t}u(t),\; t > 0$$

SOL 6.22 Option (A) is correct.

Given transfer function

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

Input $\quad r(t) = \delta(t-1)$

$$R(s) = \mathcal{L}[\delta(t-1)] = e^{-s}$$

Output is given by

$$Y(s) = R(s)\,G(s) = \frac{e^{-s}}{s^2 + 3s + 2}$$

Steady state value of output

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\frac{se^{-s}}{s^2 + 3s + 2} = 0$$

SOL 6.23 Option (A) is correct.

For C_1 Phase is given by

$$\theta_{C_1} = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$= \tan^{-1}\left(\frac{\omega - \frac{\omega}{10}}{1 + \frac{\omega^2}{10}}\right) = \tan^{-1}\left(\frac{9\omega}{10 + \omega^2}\right) > 0 \text{ (Phase lead)}$$

Similarly for C_2, phase is

$$\theta_{C2} = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}(\omega)$$

$$= \tan^{-1}\left(\frac{\frac{\omega}{10} - \omega}{1 + \frac{\omega^2}{10}}\right) = \tan^{-1}\left(\frac{-9\omega}{10 + \omega^2}\right) < 0 \text{ (Phase lag)}$$

SOL 6.24 Option (C) is correct.

From the given bode plot we can analyze that:

1. Slope -40 dB/decade→2 poles
2. Slope -20 dB/decade (Slope changes by $+20$ dB/decade)→1 Zero
3. Slope 0 dB/decade (Slope changes by $+20$ dB/decade)→1 Zero

So there are 2 poles and 2 zeroes in the transfer function.

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SOL 6.25 Option (C) is correct.

Characteristic equation for the system

$$1 + \frac{K}{s(s+3)(s+10)} = 0$$

$$s(s+3)(s+10) + K = 0$$

$$s^3 + 13s^2 + 30s + K = 0$$

Applying Routh's stability criteria

s^3	1	30
s^2	13	K
s^1	$\frac{(13 \times 30) - K}{13}$	
s^0	K	

For stability there should be no sign change in first column

So, $390 - K > 0 \Rightarrow K < 390$

$$K > 0$$

$$0 < K < 90$$

SOL 6.26 Option (C) is correct.

Given transfer function is

$$H(s)) = \frac{100}{s^2 + 20s + 100}$$

Characteristic equation of the system is given by

$$s^2 + 20s + 100 = 0$$

$$\omega_n^2 = 100 \Rightarrow \omega_n = 10 \text{ rad/sec.}$$

$$2\xi\omega_n = 20$$

or $$\xi = \frac{20}{2 \times 10} = 1$$

$(\xi = 1)$ so system is critically damped.

SOL 6.27 Option (D) is correct.

State space equation of the system is given by,

$$\dot{\boldsymbol{X}} = A\boldsymbol{X} + B\boldsymbol{u}$$

$$\boldsymbol{Y} = C\boldsymbol{X}$$

Taking Laplace transform on both sides of the equations.

$$s\boldsymbol{X}(s) = A\boldsymbol{X}(s) + B\boldsymbol{U}(s)$$

$$(sI - A)\boldsymbol{X}(s) = B\boldsymbol{U}(s)$$

$$\boldsymbol{X}(s) = (sI - A)^{-1}B\boldsymbol{U}(s)$$

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$$\therefore \quad \boldsymbol{Y}(s) = C\boldsymbol{X}(s)$$

So $$\boldsymbol{Y}(s) = C(sI - A)^{-1}B\boldsymbol{U}(s)$$

$$\text{T.F} = \frac{\boldsymbol{Y}(s)}{\boldsymbol{U}(s)} = C(sI - A)^{-1}B$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}$$

Transfer function

$$G(s) = C[sI - A]^{-1}B = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{s(s+2)} \\ \frac{1}{(s+2)} \end{bmatrix}$$

$$= \frac{1}{s(s+2)}$$

SOL 6.28 Option (A) is correct.

Steady state error is given by,

$$e_{ss} = \lim_{s \to 0}\left[\frac{sR(s)}{1 + G(s)H(s)}\right]$$

Here $$R(s) = \mathcal{L}[r(t)] = \frac{1}{s} \text{ (Unit step input)}$$

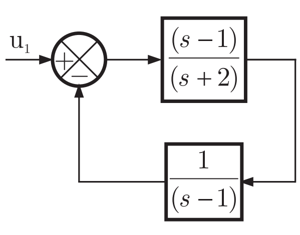
$$G(s) = \frac{1}{s(s+2)}$$

$$H(s) = 1 \text{ (Unity feed back)}$$

So, $$e_{ss} = \lim_{s \to 0}\left[\frac{s\left(\frac{1}{s}\right)}{1 + \frac{1}{s(s+2)}}\right] = \lim_{s \to 0}\left[\frac{s(s+2)}{s(s+2)+1}\right] = 0$$

SOL 6.29 Option (D) is correct.

For input u_1, the system is $(u_2 = 0)$



System response is

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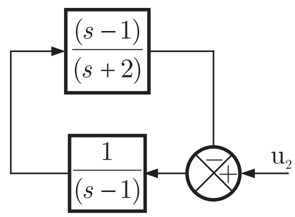
$$H_1(s) = \frac{\frac{(s-1)}{(s+2)}}{1+\frac{(s-1)}{(s+2)}\frac{1}{(s-1)}} = \frac{(s-1)}{(s+3)}$$

Poles of the system is lying at $s = -3$ (negative s-plane) so this is stable.
For input u_2 the system is $(u_1 = 0)$



System response is

$$H_2(s) = \frac{\frac{1}{(s-1)}}{1+\frac{1}{(s-1)}\frac{(s-1)}{(s+2)}} = \frac{(s+2)}{(s-1)(s+3)}$$

One pole of the system is lying in right half of s-plane, so the system is unstable.

SOL 6.30 Option (B) is correct.
Given function is.

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

By simplifying

$$G(j\omega) = \left(\frac{1}{j\omega}\times\frac{-j\omega}{-j\omega}\right)\left(\frac{1}{1+j\omega}\times\frac{1-j\omega}{1-j\omega}\right)\left(\frac{1}{2+j\omega}\times\frac{2-j\omega}{2-j\omega}\right)$$

$$= \left(-\frac{j\omega}{\omega^2}\right)\left(\frac{1-j\omega}{1+\omega^2}\right)\left(\frac{2-j\omega}{4+\omega^2}\right) = \frac{-j\omega(2-\omega^2-j3\omega)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-3\omega^2}{\omega^2(1+\omega^2)(4+\omega^2)} + \frac{j\omega(\omega^2-2)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

$$G(j\omega) = x + iy$$

$$x = \mathrm{Re}[G(j\omega)]\big|_{\omega\to 0^+} = \frac{-3}{1\times 4} = -\frac{3}{4}$$

SOL 6.31 Option (D) is correct.
Let response of the un-compensated system is

$$H_{UC}(s) = \frac{900}{s(s+1)(s+9)}$$

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Response of compensated system.

$$H_C(s) = \frac{900}{s(s+1)(s+9)} G_C(s)$$

Where $G_C(s) \to$ Response of compensator

Given that gain-crossover frequency of compensated system is same as phase crossover frequency of un-compensated system

So,

$$(\omega_g)_{\text{compensated}} = (\omega_p)_{\text{uncompensated}}$$

$$-180^\circ = \angle H_{UC}(j\omega_p)$$

$$-180^\circ = -90^\circ - \tan^{-1}(\omega_p) - \tan^{-1}\left(\frac{\omega_p}{9}\right)$$

$$90^\circ = \tan^{-1}\left(\frac{\omega_p + \frac{\omega_p}{9}}{1 - \frac{\omega_p^2}{9}}\right)$$

$$1 - \frac{\omega_p^2}{9} = 0$$

$$\omega_p = 3 \text{ rad/sec.}$$

So,

$$(\omega_g)_{\text{compensated}} = 3 \text{ rad/sec.}$$

At this frequency phase margin of compensated system is

$$\phi_{PM} = 180^\circ + \angle H_C(j\omega_g)$$

$$45^\circ = 180^\circ - 90^\circ - \tan^{-1}(\omega_g) - \tan^{-1}(\omega_g/9) + \angle G_C(j\omega_g)$$

$$45^\circ = 180^\circ - 90^\circ - \tan^{-1}(3) - \tan^{-1}(1/3) + \angle G_C(j\omega_g)$$

$$45^\circ = 90^\circ - \tan^{-1}\left[\frac{3 + \frac{1}{3}}{1 - 3\left(\frac{1}{3}\right)}\right] + \angle G_C(j\omega_g)$$

$$45^\circ = 90^\circ - 90^\circ + \angle G_C(j\omega_g)$$

$$\angle G_C(j\omega_g) = 45^\circ$$

The gain cross over frequency of compensated system is lower than un-compensated system, so we may use lag-lead compensator.

At gain cross over frequency gain of compensated system is unity so.

$$|H_C(j\omega_g)| = 1$$

$$\frac{900|G_C(j\omega_g)|}{\omega_g\sqrt{\omega_g^2+1}\sqrt{\omega_g^2+81}} = 1$$

$$|G_C(j\omega_g)| = \frac{3\sqrt{9+1}\sqrt{9+81}}{900} = \frac{3\times 30}{900} = \frac{1}{10}$$

$$\text{in dB } |G_C(\omega_g)| = 20\log\left(\frac{1}{10}\right)$$

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$$= -20 \text{ dB (attenuation)}$$

SOL 6.32 Option (B) is correct.

Characteristic equation for the given system,

$$1 + \frac{K(s+3)}{(s+8)^2} = 0$$

$$(s+8)^2 + K(s+3) = 0$$

$$s^2 + (16+K)s + (64+3K) = 0$$

By applying Routh's criteria.

s^2	1	$64+3K$
s^1	$16+K$	0
s^0	$64+3K$	

For system to be oscillatory

$$16 + K = 0 \Rightarrow K = -16$$

$$\text{Auxiliary equation } A(s) = s^2 + (64+3K) = 0$$

$$\Rightarrow \quad s^2 + 64 + 3\times(-16) = 0$$

$$s^2 + 64 - 48 = 0$$

$$s^2 = -16 \Rightarrow j\omega = 4j$$

$$\omega = 4 \text{ rad/sec}$$

SOL 6.33 Option (D) is correct.

From the given block diagram we can obtain signal flow graph of the system.

Transfer function from the signal flow graph is written as

$$\text{T.F} = \frac{\frac{c_0P}{s^2} + \frac{c_1P}{s}}{1 + \frac{a_1}{s} + \frac{a_0}{s^2} - \frac{Pb_0}{s^2} - \frac{Pb_1}{s}} = \frac{(c_0 + c_1s)P}{(s^2 + a_1s + a_0) - P(b_0 + sb_1)}$$

$$= \frac{\frac{(c_0 + c_1s)P}{(s^2 + a_1s + a_0)}}{1 - \frac{P(b_0 + sb_1)}{s^2 + a_1s + a_0}}$$

from the given reduced form transfer function is given by

$$\text{T.F} = \frac{XYP}{1 - YPZ}$$

by comparing above two we have

$$X = (c_0 + c_1s)$$

$$Y = \frac{1}{s^2 + a_1s + a_0}$$

$$Z = (b_0 + sb_1)$$

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SOL 6.34 Option (A) is correct.

For the given system Z is given by

$$Z = E(s)\frac{K_i}{s}$$

Where $E(s) \to$ steady state error of the system

Here

$$E(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

Input $\quad R(s) = \frac{1}{s}$ (Unit step)

$$G(s) = \left(\frac{K_i}{s} + K_p\right)\left(\frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}\right)$$

$$H(s) = 1 \text{ (Unity feed back)}$$

So,

$$Z = \lim_{s \to 0}\left[\frac{s\left(\frac{1}{s}\right)}{1 + \left(\frac{K_i}{s} + K_p\right)\frac{\omega^2}{(s^2 + 2\xi\omega s + \omega^2)}}\right]\left(\frac{K_i}{s}\right)$$

$$= \lim_{s \to 0}\left[\frac{K_i}{s + (K_i + K_p s)\frac{\omega^2}{(s^2 + 2\xi\omega s + \omega^2)}}\right] = \frac{K_i}{K_i} = 1$$

SOL 6.35 Option (C) is correct.

System response of the given circuit can be obtained as.

$$H(s) = \frac{e_0(s)}{e_i(s)} = \frac{\left(\frac{1}{Cs}\right)}{\left(R + Ls + \frac{1}{Cs}\right)}$$

$$H(s) = \frac{1}{LCs^2 + RCs + 1} = \frac{\left(\frac{1}{LC}\right)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Characteristic equation is given by,

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Here natural frequency $\omega_n = \frac{1}{\sqrt{LC}}$

$$2\xi\omega_n = \frac{R}{L}$$

Damping ratio $\quad \xi = \frac{R}{2L}\sqrt{LC} = \frac{R}{2}\sqrt{\frac{C}{L}}$

Here $\quad \xi = \frac{10}{2}\sqrt{\frac{1 \times 10^{-3}}{10 \times 10^{-6}}} = 0.5$ (under damped)

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So peak overshoot is given by

$$\%\text{ peak overshoot } = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100 = e^{\frac{-\pi \times 0.5}{\sqrt{1-(0.5)^2}}} \times 100 = 16\%$$

SOL 6.36 Option ( ) is correct.

SOL 6.37 Option (B) is correct.

In standard form for a characteristic equation give as

$$s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 = 0$$

in its state variable representation matrix A is given as

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}$$

Characteristic equation of the system is

$$4s^2 - 2s + 1 = 0$$

So, $a_2 = 4, a_1 = -2, a_0 = 1$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$$

SOL 6.38 Option (A) is correct.

In the given options only in option (A) the nyquist plot does not enclose the unit circle $(-1, j0)$, So this is stable.

SOL 6.39 Option (A) is correct.

Given function is,

$$H(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$$

Function can be rewritten as,

$$H(j\omega) = \frac{10^4(1+j\omega)}{10\left[1+j\frac{\omega}{10}\right]10^4\left[1+j\frac{\omega}{100}\right]^2} = \frac{0.1(1+j\omega)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{100}\right)^2}$$

The system is type 0, So, initial slope of the bode plot is 0 dB/decade.

Corner frequencies are

$$\omega_1 = 1 \text{ rad/sec}$$
$$\omega_2 = 10 \text{ rad/sec}$$
$$\omega_3 = 100 \text{ rad/sec}$$

As the initial slope of bode plot is 0 dB/decade and corner frequency $\omega_1 = 1$

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rad/sec, the Slope after $\omega = 1$ rad/sec or $\log\omega = 0$ is$(0+20) = +20$ dB/dec. After corner frequency $\omega_2 = 10$ rad/sec or $\log\omega_2 = 1$, the Slope is $(+20-20) = 0$ dB/dec.
Similarly after $\omega_3 = 100$ rad/sec or $\log\omega = 2$, the slope of plot is $(0-20\times 2) = -40$ dB/dec.
Hence (A) is correct option.

SOL 6.40 Option (B) is correct.
Given characteristic equation.

$$(s^2-4)(s+1) + K(s-1) = 0$$

or

$$1 + \frac{K(s-1)}{(s^2-4)(s+1)} = 0$$

So, the open loop transfer function for the system.

$$G(s) = \frac{K(s-1)}{(s-2)(s+2)(s+1)}, \qquad \text{no. of poles } n = 3$$

no of zeroes $m = 1$

Steps for plotting the root-locus
(1) Root loci starts at $s = 2, s = -1, s = -2$
(2) $n > m$, therefore, number of branches of root locus $b = 3$
(3) Angle of asymptotes is given by

$$\frac{(2q+1)180^\circ}{n-m}, q = 0, 1$$

(I) $\dfrac{(2\times 0+1)180^\circ}{(3-1)} = 90^\circ$

(II) $\dfrac{(2\times 1+1)180^\circ}{(3-1)} = 270^\circ$

(4) The two asymptotes intersect on real axis at

$$x = \frac{\sum\text{Poles} - \sum\text{Zeroes}}{n-m} = \frac{(-1-2+2)-(1)}{3-1} = -1$$

(5) Between two open-loop poles $s = -1$ and $s = -2$ there exist a break away point.

$$K = -\frac{(s^2-4)(s+1)}{(s-1)}$$

$$\frac{dK}{ds} = 0$$

$$s = -1.5$$

SOL 6.41 Option (C) is correct.
Closed loop transfer function of the given system is,

$$T(s) = \frac{s^2+4}{(s+1)(s+4)}$$

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$$T(j\omega) = \frac{(j\omega)^2 + 4}{(j\omega+1)(j\omega+4)}$$

If system output is zero

$$|T(j\omega)| = \frac{|4-\omega^2|}{|(j\omega+1)(j\omega+4)|} = 0$$

$$4 - \omega^2 = 0$$

$$\omega^2 = 4$$

$$\Rightarrow \omega = 2 \text{ rad/sec}$$

SOL 6.42 Option (A) is correct.

From the given plot we can see that centroid C (point of intersection) where asymptotes intersect on real axis) is 0

So for option (a)

$$G(s) = \frac{K}{s^3}$$

$$\text{Centroid} = \frac{\sum \text{Poles} - \sum \text{Zeros}}{n-m} = \frac{0-0}{3-0} = 0$$

SOL 6.43 Option (A) is correct.

Open loop transfer function is.

$$G(s) = \frac{(s+1)}{s^2}$$

$$G(j\omega) = \frac{j\omega+1}{-\omega^2}$$

Phase crossover frequency can be calculated as.

$$\angle G(j\omega_p) = -180^\circ$$

$$\tan^{-1}(\omega_p) = -180^\circ$$

$$\omega_p = 0$$

Gain margin of the system is.

$$\text{G.M} = \frac{1}{|G(j\omega_p)|} = \frac{1}{\frac{\sqrt{\omega_p^2+1}}{\omega_p^2}} = \frac{\omega_p^2}{\sqrt{\omega_p^2+1}} = 0$$

SOL 6.44 Option (C) is correct.

Characteristic equation for the given system

$$1 + G(s)H(s) = 0$$

$$1 + K\frac{(1-s)}{(1+s)} = 0$$

$$(1+s) + K(1-s) = 0$$

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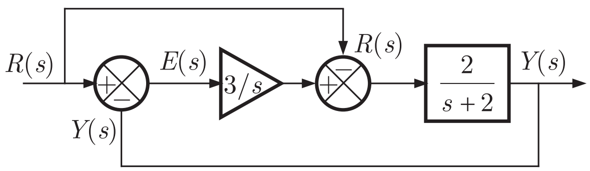
$$s(1-K)+(1+K)=0$$

For the system to be stable, coefficient of characteristic equation should be of same sign.

$$1-K>0,\ K+1>0$$
$$K<1,\ K>-1$$
$$-1<K<1$$
$$|K|<1$$

SOL 6.45 Option (C) is correct.

In the given block diagram



Steady state error is given as

$$e_{ss}=\lim_{s\to 0} sE(s)$$

$$E(s)=R(s)-Y(s)$$

$Y(s)$ can be written as

$$Y(s)=\left[\{R(s)-Y(s)\}\frac{3}{s}-R(s)\right]\frac{2}{s+2}$$

$$=R(s)\left[\frac{6}{s(s+2)}-\frac{2}{s+2}\right]-Y(s)\left[\frac{6}{s(s+2)}\right]$$

$$Y(s)\left[1+\frac{6}{s(s+2)}\right]=R(s)\left[\frac{6-2s}{s(s+2)}\right]$$

$$Y(s)=R(s)\frac{(6-2s)}{(s^2+2s+6)}$$

So,
$$E(s)=R(s)-\frac{(6-2s)}{(s^2+2s+6)}R(s)$$

$$=R(s)\left[\frac{s^2+4s}{s^2+2s+6}\right]$$

For unit step input $R(s)=\frac{1}{s}$

Steady state error $e_{ss}=\lim_{s\to 0} sE(s)$

$$e_{ss}=\lim_{s\to 0}\left[s\frac{1}{s}\frac{(s^2+4s)}{(s^2+2s+6)}\right]=0$$

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SOL 6.46 Option (B) is correct.

When it passes through negative real axis at that point phase angle is -180°

So $\angle G(j\omega)H(j\omega) = -180^\circ$

$$-0.25j\omega - \frac{\pi}{2} = -\pi$$

$$-0.25j\omega = -\frac{\pi}{2}$$

$$j0.25\omega = \frac{\pi}{2}$$

$$j\omega = \frac{\pi}{2\times 0.25}$$

$$s = j\omega = 2\pi$$

Put $s = 2\pi$ in given open loop transfer function we get

$$G(s)H(s)\big|_{s=2\pi} = \frac{\pi e^{-0.25\times 2\pi}}{2\pi} = -0.5$$

So it passes through $(-0.5, j0)$

SOL 6.47 Option (C) is correct.

Open loop transfer function of the system is given by.

$$G(s)H(s) = (K + 0.366s)\left[\frac{1}{s(s+1)}\right]$$

$$G(j\omega)H(j\omega) = \frac{K + j0.366\omega}{j\omega(j\omega+1)}$$

Phase margin of the system is given as

$$\phi_{PM} = 60^\circ = 180^\circ + \angle G(j\omega_g)H(j\omega_g)$$

Where $\omega_g \rightarrow$ gain cross over frequency $= 1$ rad/sec

So,
$$60^\circ = 180^\circ + \tan^{-1}\left(\frac{0.366\omega_g}{K}\right) - 90^\circ - \tan^{-1}(\omega_g)$$

$$= 90^\circ + \tan^{-1}\left(\frac{0.366}{K}\right) - \tan^{-1}(1)$$

$$= 90^\circ - 45^\circ + \tan^{-1}\left(\frac{0.366}{K}\right)$$

$$15^\circ = \tan^{-1}\left(\frac{0.366}{K}\right)$$

$$\frac{0.366}{K} = \tan 15^\circ$$

$$K = \frac{0.366}{0.267} = 1.366$$

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SOL 6.48 Option (A) is correct.

Given state equation.

$$\dot{\boldsymbol{X}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}\boldsymbol{X}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\boldsymbol{u}(t)$$

Here

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

State transition matrix is given by,

$$\phi(t) = \mathcal{L}^{-1}[(sI - A)^{-1}]$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s(s+3)}\begin{bmatrix} s+3 & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{(s+3)} \end{bmatrix}$$

$$\phi(t) = \mathcal{L}^{-1}[(sI - A)^{-1}]$$

$$= \begin{bmatrix} 1 & \frac{1}{3}(1 - e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$$

SOL 6.49 Option (C) is correct.

State transition equation is given by

$$\boldsymbol{X}(s) = \boldsymbol{\Phi}(s)\boldsymbol{X}(0) + \boldsymbol{\Phi}(s)B\boldsymbol{U}(s)$$

Here $\boldsymbol{\Phi}(s) \rightarrow$ state transition matrix

$$\boldsymbol{\Phi}(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{(s+3)} \end{bmatrix}$$

$\boldsymbol{X}(0) \rightarrow$ initial condition

$$\boldsymbol{X}(0) = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\text{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So

$$\boldsymbol{X}(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{(s+3)} \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} \frac{1}{s} & \frac{1}{(s+3)s} \\ 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{s}$$

$$= \begin{bmatrix} -\frac{1}{s} + \frac{3}{s(s+3)} \\ 0 + \frac{3}{s+3} \end{bmatrix} + \begin{bmatrix} \frac{1}{s} \\ 0 \end{bmatrix}\frac{1}{s} = \begin{bmatrix} -\frac{1}{s+3} \\ \frac{3}{s+3} \end{bmatrix} + \begin{bmatrix} \frac{1}{s^2} \\ 0 \end{bmatrix}$$

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$$\boldsymbol{X}(s) = \begin{bmatrix} \frac{1}{s^2} - \frac{1}{s+3} \\ \frac{3}{s+3} \end{bmatrix}$$

Taking inverse Laplace transform, we get state transition equation as,

$$\boldsymbol{X}(t) = \begin{bmatrix} t - e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

SOL 6.50 Option () is correct

Phase margin of a system is the amount of additional phase lag required to bring the system to the point of instability or $(-1, j0)$

So here phase margin $= 0^\circ$

SOL 6.51 Option (D) is correct.

Given transfer function is

$$F(s) = \frac{5}{s(s^2 + 3s + 2)}$$

$$F(s) = \frac{5}{s(s+1)(s+2)}$$

By partial fraction, we get

$$F(s) = \frac{5}{2s} - \frac{5}{s+1} + \frac{5}{2(s+2)}$$

Taking inverse Laplace of $F(s)$ we have

$$f(t) = \frac{5}{2}u(t) - 5e^{-t} + \frac{5}{2}e^{-2t}$$

So, the initial value of $f(t)$ is given by

$$\lim_{t \to 0} f(t) = \frac{5}{2} - 5 + \frac{5}{2}(1) = 0$$

SOL 6.52 Option (C) is correct.

In A.C techo-meter output voltage is directly proportional to differentiation of rotor displacement.

$$e(t) \propto \frac{d}{dt}[\theta(t)]$$

$$e(t) = K_t \frac{d\theta(t)}{dt}$$

Taking Laplace tranformation on both sides of above equation

$$E(s) = K_t s\theta(s)$$

So transfer function

$$\text{T.F} = \frac{E(s)}{\theta(s)} = (K_t)s$$

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SOL 6.53 Option (B) is correct.

Given characteristic equation,

$$s^3 - 4s^2 + s + 6 = 0$$

Applying Routh's method,

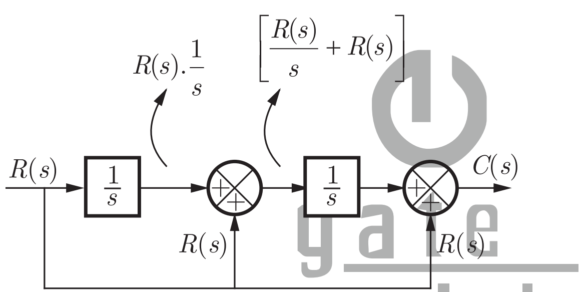
s^3	1	1
s^2	-4	6
s^1	$\frac{-4-6}{-4} = 2.5$	0
s^0	6	

There are two sign changes in the first column, so no. of right half poles is 2.

No. of roots in left half of s-plane $= (3-2) = 1$

SOL 6.54 Option (B) is correct.

Block diagram of the system is given as.



From the figure we can see that

$$C(s) = \left[R(s)\frac{1}{s} + R(s)\right]\frac{1}{s} + R(s)$$

$$C(s) = R(s)\left[\frac{1}{s^2} + \frac{1}{s} + 1\right]$$

$$\frac{C(s)}{R(s)} = \frac{1+s+s^2}{s^2}$$

SOL 6.55 Option (A) is correct.

Characteristic equation is given by,

$$|sI - A| = 0$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} s & -2 \\ -2 & s \end{bmatrix} = s^2 - 4 = 0$$

$$s_1, s_2 = \pm 2$$

SOL 6.56 Option (D) is correct.

For the given system, characteristic equation can be written as,

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$$1 + \frac{K}{s(s+2)}(1+sP) = 0$$
$$s(s+2) + K(1+sP) = 0$$
$$s^2 + s(2+KP) + K = 0$$

From the equation.

$$\omega_n = \sqrt{K} = 5 \text{ rad/sec (given)}$$

So, $K = 25$

and $2\xi\omega_n = 2 + KP$

$$2 \times 0.7 \times 5 = 2 + 25P$$

or $P = 0.2$

so $K = 25,\ P = 0.2$

SOL 6.57 Option (D) is correct.

Unit - impulse response of the system is given as,

$$c(t) = 12.5e^{-6t}\sin 8t,\ t \geq 0$$

So transfer function of the system.

$$H(s) = \mathcal{L}[c(t)] = \frac{12.5 \times 8}{(s+6)^2 + (8)^2}$$
$$H(s) = \frac{100}{s^2 + 12s + 100}$$

Steady state value of output for unit step input,

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} sH(s)R(s)$$
$$= \lim_{s \to 0} s\left[\frac{100}{s^2 + 12s + 100}\right]\frac{1}{s} = 1.0$$

SOL 6.58 Option (A) is correct.

System response is.

$$H(s) = \frac{s}{s+1}$$
$$H(j\omega) = \frac{j\omega}{j\omega + 1}$$

Amplitude response

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega + 1}}$$

Given input frequency $\omega = 1$ rad/sec.

So $\left|H(j\omega)\right|_{\omega = 1\text{ rad/sec}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

Phase response

$$\theta_h(\omega) = 90^\circ - \tan^{-1}(\omega)$$
$$\theta_h(\omega)\big|_{\omega=1} = 90^\circ - \tan^{-1}(1) = 45^\circ$$

So the output of the system is

$$y(t) = |H(j\omega)|x(t-\theta_h) = \frac{1}{\sqrt{2}}\sin(t-45^\circ)$$

SOL 6.59 Option (C) is correct.

Given open loop transfer function

$$G(j\omega) = \frac{ja\omega+1}{(j\omega)^2}$$

Gain crossover frequency (ω_g) for the system.

$$|G(j\omega_g)| = 1$$

$$\frac{\sqrt{a^2\omega_g^2+1}}{-\omega_g^2} = 1$$

$$a^2\omega_g^2+1 = \omega_g^4$$

$$\omega_g^4 - a^2\omega_g^2 - 1 = 0 \qquad ...(1)$$

Phase margin of the system is

$$\phi_{PM} = 45^\circ = 180^\circ + \angle G(j\omega_g)$$

$$45^\circ = 180^\circ + \tan^{-1}(\omega_g a) - 180^\circ$$

$$\tan^{-1}(\omega_g a) = 45^\circ$$

$$\omega_g a = 1 \qquad (2)$$

From equation (1) and (2)

$$\frac{1}{a^4} - 1 - 1 = 0$$

$$a^4 = \frac{1}{2} \Rightarrow a = 0.841$$

SOL 6.60 Option (C) is correct.

Given system equation is.

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 12(1-e^{-2t})$$

Taking Laplace transform on both side.

$$s^2X(s) + 6sX(s) + 5X(s) = 12\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$(s^2+6s+5)X(s) = 12\left[\frac{2}{s(s+2)}\right]$$

System transfer function is

$$X(s) = \frac{24}{s(s+2)(s+5)(s+1)}$$

Response of the system as $t \to \infty$ is given by

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \quad \text{(final value theorem)}$$

$$= \lim_{s \to 0} s\left[\frac{24}{s(s+2)(s+5)(s+1)}\right]$$

$$= \frac{24}{2 \times 5} = 2.4$$

SOL 6.61 Option (A) is correct.

Transfer function of lead compensator is given by.

$$H(s) = \frac{K\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)}$$

$$H(j\omega) = K\left[\frac{1+j\left(\frac{\omega}{a}\right)}{1+j\left(\frac{\omega}{b}\right)}\right]$$

So, phase response of the compensator is.

$$\theta_h(\omega) = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\omega}{a}-\frac{\omega}{b}}{1+\frac{\omega^2}{ab}}\right) = \tan^{-1}\left[\frac{\omega(b-a)}{ab+\omega^2}\right]$$

θ_h should be positive for phase lead compensation

So, $$\theta_h(\omega) = \tan^{-1}\left[\frac{\omega(b-a)}{ab+\omega^2}\right] > 0$$

$$b > a$$

SOL 6.62 Option (A) is correct.

Since there is no external input, so state is given by

$$\boldsymbol{X}(t) = \phi(t)\boldsymbol{X}(0)$$

$\phi(t)$ $\rightarrow$state transition matrix

$\boldsymbol{X}[0]$ $\rightarrow$initial condition

So $$x(t) = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 2e^{-2t} \\ 3e^{-t} \end{bmatrix}$$

At $t = 1$, state of the system

$$x(t)\big|_{t=1} = \begin{bmatrix} 2e^{-2} \\ 2e^{-1} \end{bmatrix} = \begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$$

SOL 6.63 Option (B) is correct.

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Given equation

$$\frac{d^2x}{dt^2} + \frac{1}{2}\frac{dx}{dt} + \frac{1}{18}x = 10 + 5e^{-4t} + 2e^{-5t}$$

Taking Laplace on both sides we have

$$s^2X(s) + \frac{1}{2}sX(s) + \frac{1}{18}X(s) = \frac{10}{s} + \frac{5}{s+4} + \frac{2}{s+5}$$

$$\left(s^2 + \frac{1}{2}s + \frac{1}{18}\right)X(s) = \frac{10(s+4)(s+5) + 5s(s+5) + 2s(s+4)}{s(s+4)(s+5)}$$

System response is,

$$X(s) = \frac{10(s+4)(s+5) + 5s(s+5) + 2s(s+4)}{s(s+4)(s+5)\left(s^2 + \frac{1}{2}s + \frac{1}{18}\right)}$$

$$= \frac{10(s+4)(s+5) + 5s(s+5) + 2s(s+4)}{s(s+4)(s+5)\left(s + \frac{1}{3}\right)\left(s + \frac{1}{6}\right)}$$

We know that for a system having many poles, nearness of the poles towards imaginary axis in s-plane dominates the nature of time response. So here time constant given by two poles which are nearest to imaginary axis.

Poles nearest to imaginary axis

$$s_1 = -\frac{1}{3},\ s_2 = -\frac{1}{6}$$

So, time constants $\begin{cases} \tau_1 = 3\text{ sec} \\ \tau_2 = 6\text{ sec} \end{cases}$

SOL 6.64 Option (A) is correct.

Steady state error for a system is given by

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

Where input $R(s) = \frac{1}{s}$ (unit step)

$$G(s) = \left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)$$

$$H(s) = 1 \quad \text{(unity feedback)}$$

So

$$e_{ss} = \lim_{s \to 0} \frac{s\left(\frac{1}{s}\right)}{1 + \frac{45}{(s+15)(s+1)}} = \frac{15}{15+45} = \frac{15}{60}$$

$$\% e_{ss} = \frac{15}{60} \times 100 = 25\%$$

SOL 6.65 Option (C) is correct.

Characteristic equation is given by

$$1 + G(s)H(s) = 0$$

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Here $H(s) = 1$ (unity feedback)

$$G(s) = \left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)$$

So, $1 + \left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right) = 0$

$$(s+15)(s+1) + 45 = 0$$
$$s^2 + 16s + 60 = 0$$
$$(s+6)(s+10) = 0$$
$$s = -6, -10$$

SOL 6.66 Option (A) is correct.

Given equation can be written as,

$$\frac{d^2\omega}{dt^2} = -\frac{\beta}{J}\frac{d\omega}{dt} - \frac{K^2}{LJ}\omega + \frac{K}{LJ}V_a$$

Here state variables are defined as,

$$\frac{d\omega}{dt} = x_1$$
$$\omega = x_2$$

So state equation is

$$\dot{x}_1 = -\frac{B}{J}x_1 - \frac{K^2}{LJ}x_2 + \frac{K}{LJ}V_a$$
$$\dot{x}_2 = \frac{d\omega}{dt} = x_1$$

In matrix form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-B/J & -K^2/LJ\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}K/LJ\\ 0\end{bmatrix}V_a$$

$$\begin{bmatrix}\frac{d^2\omega}{dt^2}\\ \frac{d\omega}{dt}\end{bmatrix} = P\begin{bmatrix}\frac{d\omega}{dt}\end{bmatrix} + QV_a$$

So matrix P is

$$\begin{bmatrix}-B/J & -K^2/LJ\\ 1 & 0\end{bmatrix}$$

SOL 6.67 Option (C) is correct.

Characteristic equation of the system is given by

$$1 + GH = 0$$
$$1 + \frac{K}{s(s+2)(s+4)} = 0$$
$$s(s+2)(s+4) + K = 0$$
$$s^3 + 6s^2 + 8s + K = 0$$

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Applying routh's criteria for stability

s^3	1	8
s^2	6	K
s^1	$\frac{K-48}{6}$	
s^0	K	

System becomes unstable if $\frac{K-48}{6} = 0 \Rightarrow K = 48$

SOL 6.68 Option (A) is correct.

The maximum error between the exact and asymptotic plot occurs at corner frequency.

Here exact gain(dB) at $\omega = 0.5a$ is given by

$$\text{gain(dB)}\Big|_{\omega = 0.5a} = 20\log K - 20\log\sqrt{1 + \frac{\omega^2}{a^2}}$$

$$= 20\log K - 20\log\left[1 + \frac{(0.5a)^2}{a^2}\right]^{1/2} = 20\log K - 0.96$$

Gain(dB) calculated from asymptotic plot at $\omega = 0.5a$ is

$$= 20\log K$$

Error in gain (dB) $= 20\log K - (20\log K - 0.96)\text{ dB} = 0.96\text{ dB}$

Similarly exact phase angle at $\omega = 0.5a$ is.

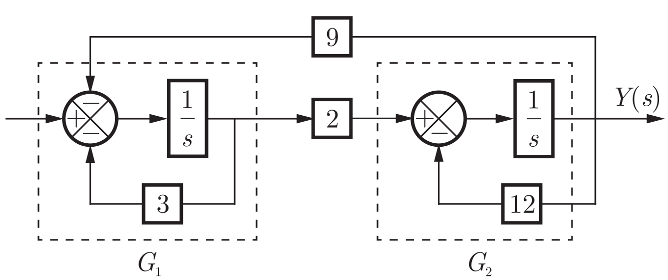
$$\theta_h(\omega)\Big|_{\omega = 0.5a} = -\tan^{-1}\left(\frac{\omega}{a}\right) = -\tan^{-1}\left(\frac{0.5a}{a}\right) = -26.56^\circ$$

Phase angle calculated from asymptotic plot at $(\omega = 0.5a)$ is -22.5°

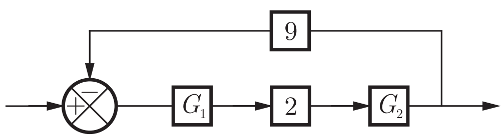
Error in phase angle $= -22.5 - (-26.56^\circ) = 4.9^\circ$

SOL 6.69 Option (B) is correct.

Given block diagram



Given block diagram can be reduced as



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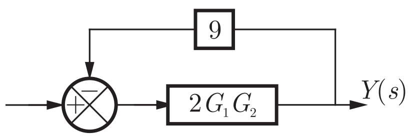
Where $$G_1 = \frac{\left(\frac{1}{s}\right)}{1+\left(\frac{1}{s}\right)^3} = \frac{1}{s+3}$$

$$G_2 = \frac{\left(\frac{1}{s}\right)}{1+\left(\frac{1}{s}\right)12} = \frac{1}{s+12}$$

Further reducing the block diagram.



$$Y(s) = \frac{2G_1G_2}{1+(2G_1G_2)9}$$

$$= \frac{(2)\left(\frac{1}{s+3}\right)\left(\frac{1}{s+12}\right)}{1+(2)\left(\frac{1}{s+3}\right)\left(\frac{1}{s+12}\right)(9)}$$

$$= \frac{2}{(s+3)(s+12)+18} = \frac{2}{s^2+15s+54}$$

$$= \frac{2}{(s+9)(s+6)} = \frac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{6}\right)}$$

SOL 6.70 Option (C) is correct.

Given state equation is,

$$\dot{\boldsymbol{X}} = A\boldsymbol{X}$$

Taking Laplace transform on both sides of the equation,

$$s\boldsymbol{X}(s) - \boldsymbol{X}(0) = A\boldsymbol{X}(s)$$

$$(sI-A)\boldsymbol{X}(s) = \boldsymbol{X}(0)$$

$$\boldsymbol{X}(s) = (sI-A)^{-1}\boldsymbol{X}(0) = \Phi(s)\boldsymbol{X}(0)$$

Where $\phi(t) = \mathcal{L}^{-1}[\boldsymbol{\Phi}(s)] = \mathcal{L}^{-1}[(sI-A)^{-1}]$ is defined as state transition matrix

SOL 6.71 Option (B) is correct.

State equation of the system is given as,

$$\dot{\boldsymbol{X}} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}\boldsymbol{X} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\boldsymbol{u}$$

Here $$A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Check for controllability:

$$AB = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$U = [B : AB] = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$|U| = (1 \times 0 - 2 \times 0) = 0$$

Matrix U is singular, so the system is uncontrollable.

Check for Stability:

Characteristic equation of the system is obtained as,

$$|sI - A| = 0$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} s-2 & -3 \\ 0 & s-5 \end{bmatrix}$$

$$|sI - A| = (s-2)(s-5) = 0$$

$$s = 2,\ s = 5$$

There are two R.H.S Poles in the system so it is unstable.

SOL 6.72 Option (B) is correct.

Given open loop transfer function,

$$G(s) = \frac{K}{s^2}, \quad \begin{aligned} &\text{no of poles} = 2 \\ &\text{no of zeroes} = 0 \end{aligned}$$

For plotting root locus:

(1) Poles lie at $s_1, s_2 = 0$

(2) So the root loci starts $(K = 0)$ from s = 0 and s = 0

(3) As there is no open-loop zero, root loci terminates $(K = \infty)$ at infinity.

(4) Angle of asymptotes is given by

$$\frac{(2q+1)180^\circ}{n-m}, \ q = 0, 1$$

So the two asymptotes are at an angle of

(i) $\frac{(2 \times 0 + 1)180^\circ}{2} = 90^\circ$

(ii) $\frac{(2 \times 1 + 1)180^\circ}{2} = 270^\circ$

(5) The asymptotes intersect on real axis at a point given by

$$x = \frac{\sum \text{Poles} - \sum \text{zeros}}{n-m} = \frac{0-0}{2} = 0$$

(6) Break away points

$$1 + \frac{K}{s^2} = 0$$

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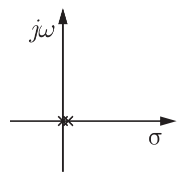
$$K = -s^2$$
$$\frac{dK}{ds} = -2s = 0 \Rightarrow s = 0$$

So the root locus plot is.



SOL 6.73 Option (A) is correct.

System is described as.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} = \frac{du}{dt} + 2u$$

Taking Laplace transform on both sides.

$$s^2 Y(s) + sY(s) = sU(s) + 2U(s)$$
$$(s^2 + s)\,Y(s) = (s+2)\,U(s)$$

So, the transfer function is

$$\text{T.F} = \frac{Y(s)}{U(s)} = \frac{(s+2)}{(s^2+s)}$$

SOL 6.74 Option (A) is correct.

Here, we have

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C = [4,\ 0]$$

We know that transfer function of the system is given by.

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} s-2 & 0 \\ 0 & s-4 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s-2)(s-4)}\begin{bmatrix} (s-4) & 0 \\ 0 & (s-2) \end{bmatrix} = \begin{bmatrix} \frac{1}{(s-2)} & 0 \\ 0 & \frac{1}{(s-4)} \end{bmatrix}$$

So,
$$\frac{Y(s)}{U(s)} = [4\ \ 0]\begin{bmatrix} \frac{1}{(s-2)} & 0 \\ 0 & \frac{1}{(s-4)} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = [4\ \ 0]\begin{bmatrix} \frac{1}{(s-2)} \\ \frac{1}{(s-4)} \end{bmatrix}$$

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$$\frac{Y(s)}{U(s)} = \frac{4}{(s-2)}$$

Here input is unit impulse so $U(s) = 1$ and output

$$Y(s) = \frac{4}{(s-2)}$$

Taking inverse Laplace transfer we get output

$$y(t) = 4e^{2t}$$

SOL 6.75 Option (D) is correct.

Given state equation

$$\dot{\boldsymbol{X}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{X}$$

Here $$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Eigen value can be obtained as

$$|A - \lambda I| = 0$$

$$(A - \lambda I) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & 0 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \lambda^3(1-\lambda) = 0$$

or $\lambda_1, \lambda_2, \lambda_3 = 0, \lambda_4 = 1$

SOL 6.76 Option (A) is correct.

Input-output relationship is given as

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 5\frac{du}{dt} - 3u$$

Taking Laplace transform on both sides with zero initial condition.

$$s^2Y(s) + 2sY(s) + 10Y(s) = 5sU(s) - 3U(s)$$

$$(s^2 + 2s + 10)\,Y(s) = (5s-3)\,U(s)$$

Output $$Y(s) = \frac{(5s-3)}{(s^2+2s+10)}U(s)$$

With no input and with given initial conditions, output is obtained as

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 0$$

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Taking Laplace transform (with initial conditions)

$[s^2 Y(s) - sy(0) - y'(0)] + 2[sY(s) - y(0)] + 10Y(s) = 0$

Given that $y'(0) = -4,\ y(0) = 1$

$$[s^2 Y(s) - s - (-4)] + 2(s-1) + 10Y(s) = 0$$

$$Y(s)[s^2 + 2s + 10] = (s-2)$$

$$Y(s) = \frac{(s-2)}{(s^2+2s+10)}$$

Output in both cases are same so

$$\frac{(5s-3)}{(s^2+2s+10)}U(s) = \frac{(s-2)}{(s^2+2s+10)}$$

$$U(s) = \frac{(s-2)}{(5s-3)} = \frac{1}{5}\frac{(5s-10)}{(5s-3)}$$

$$= \frac{1}{5}\left[\frac{(5s-3)}{5s-3} - \frac{7}{(5s-3)}\right]$$

$$U(s) = \frac{1}{5}\left[1 - \frac{7}{(5s-3)}\right]$$

Taking inverse Laplace transform, input is

$$u(t) = \frac{1}{5}\left[\delta(t) - \frac{5}{5}e^{3/5t}u(t)\right] = \frac{1}{5}\delta(t) - \frac{7}{25}e^{3/5t}u(t)$$

SOL 6.77 Option (C) is correct.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = u(t)e^{-t} \qquad ...(1)$$

State variable representation is given as

$$\dot{\boldsymbol{X}} = A\boldsymbol{X} + B\boldsymbol{u}$$

Or $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = A\begin{bmatrix}x_1\\ x_2\end{bmatrix} + B\boldsymbol{u}$$

Here $x_1 = y,\ x_2 = \left(\frac{dy}{dt} - y\right)e^t$

$$\frac{dx_1}{dt} = \frac{dy}{dt} = x_2 e^{-t} + y = x_2 e^{-t} + x_1$$

or $$\frac{dx_1}{dt} = x_1 + x_2 e^{-t} + (0)u(t) \qquad ...(2)$$

Similarly

$$\frac{dx_2}{dt} = \frac{d^2y}{dt^2}e^t + \frac{dy}{dt}e^t - e^t\frac{dy}{dt} - ye^t$$

Put $\frac{d^2y}{dt^2}$ from equation (1)

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So, $\frac{dx_2}{dt} = \left[u(t)\,e^{-t} - \frac{dy}{dt} + 2y\right]e^t - ye^t$

$$= u(t) - \frac{dy}{dt}e^t + 2ye^t - ye^t = u(t) - [x_2 e^{-t} + y]\,e^t + ye^t$$

$$= u(t) - x_2$$

$$\frac{dx_2}{dt} = 0 - x_2 + u(t) \qquad ...(3)$$

From equation (2) and (3) state variable representation is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}1 & e^{-t}\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

SOL 6.78 Option (B) is correct.

Characteristic equation of the system

$$1 + G(s) = 0$$

$$1 + \frac{2(s+\alpha)}{s(s+2)(s+10)} = 0$$

$$s(s+2)(s+10) + 2(s+\alpha) = 0$$

$$s^3 + 12s^2 + 20s + 2s + 2\alpha = 0$$

$$s^3 + 12s^2 + 22s + 2\alpha = 0$$

$$1 + \frac{2\alpha}{s^3 + 12s^2 + 22s} = 0$$

No of poles $n = 3$

No. of zeros $m = 0$

Angle of asymptotes

$$\phi_A = \frac{(2q+1)180^\circ}{n-m}, \; q = 0,1,2$$

$$\phi_A = \frac{(2q+1)180^\circ}{3} = (2q+1)60^\circ$$

$$\phi_A = 60^\circ, 180^\circ, 300^\circ$$

SOL 6.79 Option (A) is correct.

Asymptotes intercepts at real axis at the point

$$C = \frac{\sum \text{real Parts of Poles} - \sum \text{real Parts of zeros}}{n-m}$$

Poles at $s_1 = 0$

$s_2 = -2$

$s_3 = -10$

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So $$C = \frac{0-2-10-0}{3-0} = -4$$

SOL 6.80 Option (C) is correct.

Break away points

$$\frac{d\alpha}{ds} = 0$$

$$\alpha = -\frac{1}{2}[s^3 + 12s^2 + 22s]$$

$$\frac{d\alpha}{ds} = -\frac{1}{2}[3s^2 + 24s + 22] = 0$$

$$s_1, s_2 = -1.056,\ -6.9433$$

SOL 6.81 Option ( ) is correct.

SOL 6.82 Option (A) is correct.

Given state equation

$$\dot{\boldsymbol{X}} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}\boldsymbol{X}$$

Or $$\dot{\boldsymbol{X}} = A\boldsymbol{X}, \text{ where } A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

Taking Laplace transform on both sides.

$$s\boldsymbol{X}(s) - \boldsymbol{X}(0) = A\boldsymbol{X}(s)$$

$$\boldsymbol{X}(s)(sI - A) = \boldsymbol{X}(0)$$

$$\boldsymbol{X}(s) = (sI - A)^{-1}\boldsymbol{X}(0)$$

Steady state value of $\boldsymbol{X}$ is given by

$$x_{ss} = \lim_{s \to 0} s\boldsymbol{X}(s) = \lim_{s \to 0} s(sI - A)^{-1}\boldsymbol{X}(0)$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$(sI - A^{-1}) = \frac{1}{(s+3)(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{(s+3)} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}$$

So the steady state value

$$x_{ss} = \lim_{s \to 0} s\begin{bmatrix} \frac{1}{(s+3)} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}\begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$= \lim_{s \to 0} s \begin{bmatrix} \frac{10}{(s+3)} - \frac{10}{(s+2)(s+3)} \\ \frac{-10}{(s+2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

SOL 6.83 Option (D) is correct.

Initial slope of the bode plot is -40 dB/dec. So no. of poles at origin is 2. Then slope increased by -20 dB/dec. at $\omega = 2$ rad/sec, so one poles lies at this frequency. At $\omega = 5$ rad/sec slope changes by $+20$ dB/dec, so there is one zero lying at this frequency. Further slope decrease by -20 dB/dec at $\omega = 25$ so one pole of the system is lying at this frequency.

Transfer function

$$H(s) = \frac{K(s+5)}{s^2(s+2)(s+25)}$$

At $\omega = 0.1$, gain is 54 dB, so

$$54 = 20\log\frac{5K}{(0.1)^2(2)(25)}$$

$$K = 50$$

$$H(s) = \frac{50(s+5)}{s^2(s+2)(s+25)}$$

SOL 6.84 Option (B) is correct.

Open loop transfer function of the system is

$$G(s) = \frac{10^4}{s(s+10)^2}$$

$$G(j\omega) = \frac{10^4}{j\omega(j\omega+10)^2} = \frac{10^4}{j\omega(100-\omega^2+j20\omega)}$$

Magnitude $$|G(j\omega)| = \frac{10^4}{\omega\sqrt{(100-\omega^2)^2+400\omega^2}}$$

At $\omega = 20$ rad/sec

$$|G(j20)| = \frac{10^4}{20\sqrt{9\times10^4+16\times10^4}} = \frac{10^4}{20\times5\times10^2} = 1$$

$$\text{Magnitude in dB} = 20\log_{10}|G(j20)| = 20\log_{10}1 = 0 \text{ dB}$$

SOL 6.85 Option (C) is correct.

Since $|G(j\omega)| = 1$ at $\omega = 20$ rad/sec, So this is the gain cross-over frequency

$$\omega_g = 20 \text{ rad/sec}$$

Phase margin $$\phi_{PM} = 180^\circ + \angle G(j\omega_g)$$

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$$\angle G(j\omega_g) = -90^\circ - \tan^{-1}\left[\frac{20\,\omega_g}{100-\omega_g^2}\right]$$

$$\phi_{PM} = 180 - 90^\circ - \tan^{-1}\left[\frac{20\times 20}{100-(20)^2}\right] = -36.86^\circ$$

SOL 6.86 Option (C) is correct.

To calculate the gain margin, first we have to obtain phase cross over frequency (ω_p).

At phase cross over frequency

$$\angle G(j\omega_p) = -180^\circ$$

$$-90^\circ - \tan^{-1}\left[\frac{20\omega_p}{100-\omega_p^2}\right] = -180^\circ$$

$$\tan^{-1}\left[\frac{20\omega_p}{100-\omega_p^2}\right] = 90^\circ$$

$$100-\omega_p^2 = 0 \Rightarrow \omega_p = 10 \text{ rad/sec.}$$

$$\text{Gain margin in dB} = 20\log_{10}\left(\frac{1}{|G(j\omega_p)|}\right)$$

$$|G(j\omega_p)| = |G(j10)| = \frac{10^4}{10\sqrt{(100-100)^2+400(10)^2}}$$

$$= \frac{10^4}{10\times 2\times 10^2} = 5$$

$$\text{G.M.} = 20\log_{10}\left(\frac{1}{5}\right) = -13.97 \text{ dB}$$

SOL 6.87 Option (B) is correct.

Since gain margin and phase margin are negative, so the system is unstable.

SOL 6.88 Option (C) is correct.

Given characteristic equation

$$s^3+s^2+Ks+K = 0$$

$$1+\frac{K(s+1)}{s^3+s^2} = 0$$

$$1+\frac{K(s+1)}{s^2(s+2)} = 0$$

so open loop transfer function is

$$G(s) = \frac{K(s+1)}{s^2(s+1)}$$

root-locus is obtained in following steps:

1. Root-loci starts($K=0$) at $s=0$, $s=0$ and $s=-2$
2. There is one zero at $s=-1$, so one of root-loci terminates at $s=-1$ and other two terminates at infinity

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3. No. of poles $n = 3$, no of zeros $, m = 1$
4. Break - Away points

$$\frac{dK}{ds} = 0$$

Asymptotes meets on real axis at a point C

$$C = \frac{\sum \text{poles} - \sum \text{zeros}}{n - m}$$

$$= \frac{(0 + 0 - 2) - (-1)}{3 - 1} = -0.5$$

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GATE Multiple Choice Questions
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